4.1. Limesi

4. 12. 2020.

Fotografija: Jan Huber

“Definicija”

Neka je $f : D \subseteq \mathbb{R} \to \mathbb{R}$. Za $c \in \mathbb{R} \cup \{\pm\infty\}$, ako vrijedi

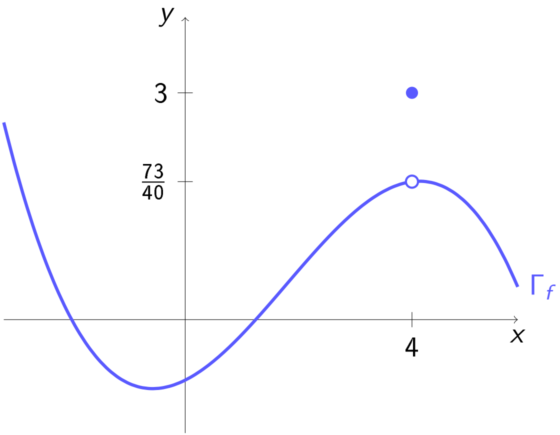
$$\underbrace{x \to c}_{\text{“}x\text{ se približava }c,\ x \neq c\text{”}} \quad \Rightarrow \quad \underbrace{f(x) \to L}_{\text{“}f(x)\text{ se približava }L\text{”}}$$

za neki $L \in \mathbb{R} \cup \{\pm\infty\}$, pišemo

$$\lim_{x \to c} f(x) = L.$$

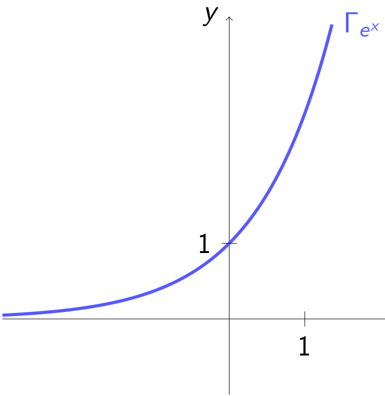
Ako je $L \in \mathbb{R}$, kažemo da je L **limes funkcije** f **u točki** c.

Primjer 1(a)



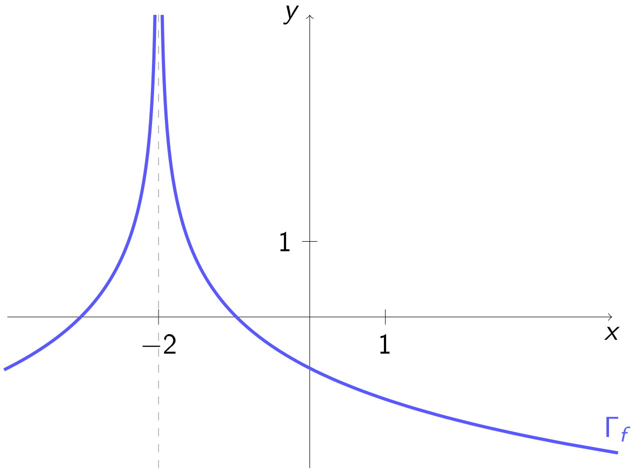
$$\lim_{x \to 4} f(x) = \frac{73}{40}.$$

Primjer 1(b)



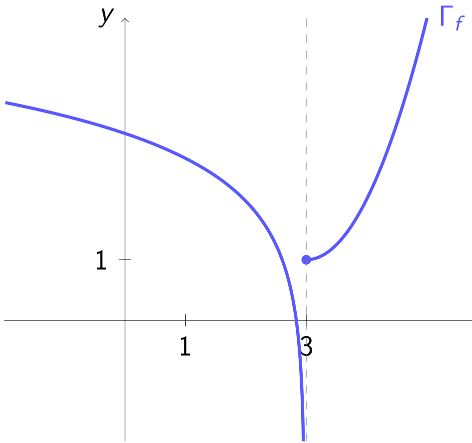
$$\lim_{x \to -\infty} e^x = 0, \qquad \lim_{x \to +\infty} e^x = +\infty, \qquad \lim_{x \to 0} e^x = 1 (= e^0).$$

Primjer 1(c)



$$\lim_{x \to -2} f(x) = +\infty.$$

Primjer 1(d)



$\lim_{x\to 3} f(x)$ ne postoji, ali $\lim_{x\to 3-} f(x) = -\infty$, a $\lim_{x\to 3+} f(x) = 1.$

“Definicija”

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$$\underbrace{x \to c+}_{\text{“}x\text{ se približava }c,\ x > c\text{”}} \quad \Rightarrow \quad \underbrace{f(x) \to L}_{\text{“}f(x)\text{ se približava }L\text{”}}$$

za neki $L \in \mathbb{R} \cup \{\pm\infty\}$, pišemo

$$\lim_{x \to c+} f(x) = L.$$

Ako je $L \in \mathbb{R}$, kažemo da je L **limes zdesna funkcije** f **u točki** c.

“Definicija”

Neka je $f : D \subseteq \mathbb{R} \to \mathbb{R}$. Za $c \in \mathbb{R}$, ako vrijedi

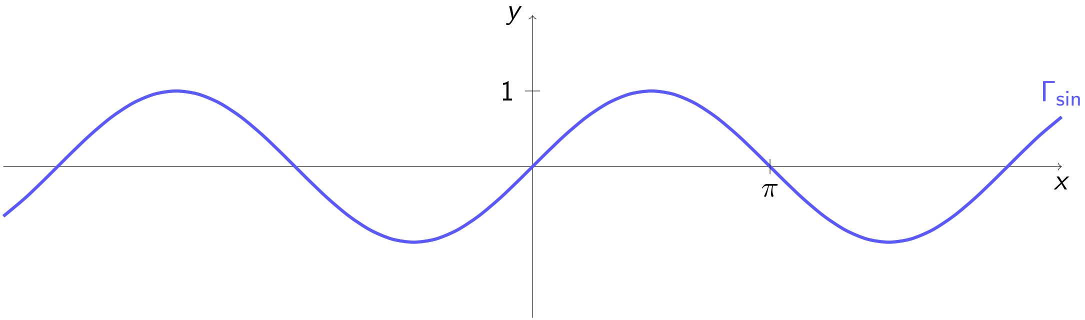
$$\underbrace{x \to c-}_{\text{“}x \text{ se približava } c,\ x<c\text{”}} \quad \Rightarrow \quad \underbrace{f(x) \to L}_{\text{“}f(x) \text{ se približava } L\text{”}}$$

za neki $L \in \mathbb{R} \cup \{\pm\infty\}$, pišemo

$$\lim_{x \to c-} f(x) = L.$$

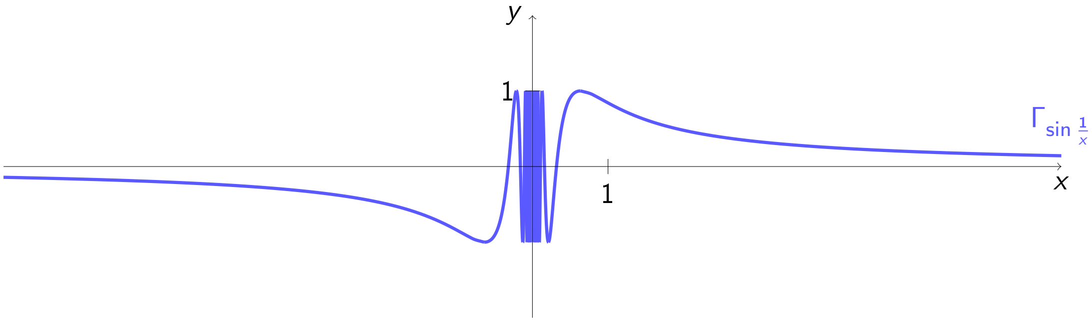
Ako je $L \in \mathbb{R}$, kažemo da je L **limes slijeva funkcije f u točki c**.

Primjer 1(e)



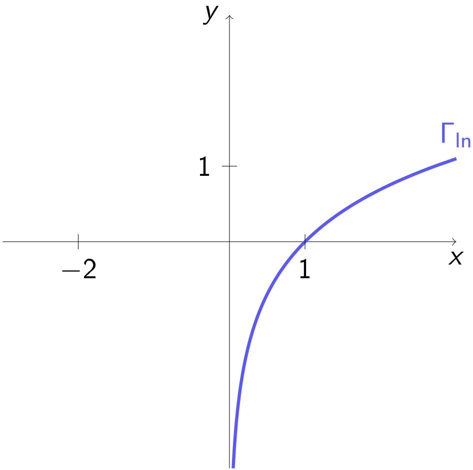
$$\lim_{x \to +\infty} \sin x \text{ ne postoji.}$$

Primjer 1(f)



$$\lim_{x\to 0+} \sin\frac{1}{x} \quad \text{i} \quad \lim_{x\to 0-} \sin\frac{1}{x} \quad \text{ne postoje.}$$

Primjer 1(g)



$$\lim_{x\to -2-} \ln x \qquad \text{i} \qquad \lim_{x\to -2+} \ln x \text{ ne postoje.}$$

Neka su $a \in \mathbb{R}$ i $c \in \mathbb{R} \cup \{\pm\infty\}$. Svaka od sljedećih jednakosti vrijedi kad god je njezina desna strana definirana:

(i) $\lim\limits_{x\to c} (f(x) + g(x)) = \lim\limits_{x\to c} f(x) + \lim\limits_{x\to c} g(x)$

(ii) $\lim\limits_{x\to c} f(x) \cdot g(x) = \lim\limits_{x\to c} f(x) \cdot \lim\limits_{x\to c} g(x)$

(iii) $\lim\limits_{x\to c} a\, f(x) = a \cdot \lim\limits_{x\to c} f(x)$

(iv) $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$.

Analogne tvrdnje vrijede i za $\lim_{x\to c+}$ i $\lim_{x\to c-}$.

Primjer 2(a)

$$\lim_{x\to+\infty} \frac{(3x+5)(2x-3)(4x-7)}{3x^3+x-1}$$

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$$\lim_{x\to+\infty}\frac{(3x+5)(2x-3)(4x-7)}{3x^3+x-1}=\left(\frac{+\infty}{+\infty}\right)$$

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$$=\lim_{x\to+\infty}\frac{(3x+5)(2x-3)(4x-7)}{3x^3+x-1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

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$$=\lim_{x\to+\infty}\frac{\frac{3x+5}{x}\cdot\frac{2x-3}{x}\cdot\frac{4x-7}{x}}{3+\frac{1}{x^2}-\frac{1}{x^3}}$$

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$$=\lim_{x\to+\infty}\frac{\left(3+\frac{5}{x}\right)\left(2-\frac{3}{x}\right)\left(4-\frac{7}{x}\right)}{3+\frac{1}{x^2}-\frac{1}{x^3}}$$

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$$\lim_{x\to+\infty} \frac{(3x+5)(2x-3)(4x-7)}{3x^3+x-1} = \left(\frac{+\infty}{+\infty}\right)$$

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$$= \frac{\left(\lim_{x\to+\infty} 3 + \lim_{x\to+\infty} \frac{5}{x}\right)\left(\lim_{x\to+\infty} 2 - \lim_{x\to+\infty} \frac{3}{x}\right)\left(\lim_{x\to+\infty} 4 - \lim_{x\to+\infty} \frac{7}{x}\right)}{\lim_{x\to+\infty} 3 + \lim_{x\to+\infty} \frac{1}{x^2} - \lim_{x\to+\infty} \frac{1}{x^3}}$$

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$$=\frac{\left(\lim_{x\to+\infty}3+\lim_{x\to+\infty}\frac{5}{x}\right)\left(\lim_{x\to+\infty}2-\lim_{x\to+\infty}\frac{3}{x}\right)\left(\lim_{x\to+\infty}4-\lim_{x\to+\infty}\frac{7}{x}\right)}{\lim_{x\to+\infty}3+\lim_{x\to+\infty}\frac{1}{x^2}-\lim_{x\to+\infty}\frac{1}{x^3}}$$

$$=\frac{(3+0)(2-0)(4-0)}{3+0-0}$$

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$$=\frac{\left(\lim_{x\to+\infty}3+\lim_{x\to+\infty}\frac{5}{x}\right)\left(\lim_{x\to+\infty}2-\lim_{x\to+\infty}\frac{3}{x}\right)\left(\lim_{x\to+\infty}4-\lim_{x\to+\infty}\frac{7}{x}\right)}{\lim_{x\to+\infty}3+\lim_{x\to+\infty}\frac{1}{x^2}-\lim_{x\to+\infty}\frac{1}{x^3}}$$

$$=\frac{(3+0)(2-0)(4-0)}{3+0-0}$$

$$=8.$$

Primjer 2(b)

$$\lim_{x \to +\infty} \frac{x}{\sqrt[3]{x^3 + 10}}$$

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$$= \lim_{x\to+\infty} \frac{1}{\frac{\sqrt[3]{x^3+10}}{\sqrt[3]{x^3}}}$$

Primjer 2(b)

$$\begin{aligned}
\lim_{x\to+\infty} \frac{x}{\sqrt[3]{x^3+10}} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{x}{\sqrt[3]{x^3+10}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to+\infty} \frac{1}{\frac{\sqrt[3]{x^3+10}}{\sqrt[3]{x^3}}} \\
&= \lim_{x\to+\infty} \frac{1}{\sqrt[3]{1+\frac{10}{x^3}}}
\end{aligned}$$

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$$= 1.$$

Primjer 2(b)

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\end{aligned}
$$

Pitanje. Smije li $\lim_{x\to+\infty}$ ući pod $\sqrt[3]{}$?

$$
\begin{aligned}
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&= \lim_{x\to+\infty} \frac{1}{\frac{\sqrt[3]{x^3+10}}{\sqrt[3]{x^3}}} \\
&= \lim_{x\to+\infty} \frac{1}{\sqrt[3]{1+\frac{10}{x^3}}} = \frac{1}{\sqrt[3]{1+\lim_{x\to+\infty}\frac{10}{x^3}}} \\
&= \frac{1}{\sqrt[3]{1+0}} \\
&= 1.
\end{aligned}
$$

Pitanje. Smije li $\lim_{x\to+\infty}$ ući pod $\sqrt[3]{}$? **Da.**

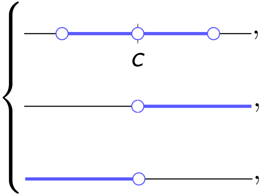
Smije li lim ući pod $\sqrt[3]{\ }$?

Vrijedi

$$\lim_{x\to c} g(f(x)) = g\left(\lim_{x\to c} f(x)\right) \tag{1}$$

kad god su zadovoljeni sljedeći uvjeti:

- g je elementarna funkcija: polinom, racionalna funkcija, $\sqrt[n]{\cdot}$, trigonometrijska, arkus, eksponencijalna, logaritamska funkcija ili $|\cdot|$.
- Desna strana formule (1) je definirana.
- $g(f(x))$ je definirano za x iz nekog **probušenog intervala oko** c, tj. intervala oblika



ako je $c \in \mathbb{R}$,

ako je $c = +\infty$,

ako je $c = -\infty$.

Primjer 3

(a) Oprez!

$$\lim_{x\to 0}\sqrt{-x^2} \neq \sqrt{\lim_{x\to 0}(-x^2)} = \sqrt{0} = 0.$$

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Zapravo, $\lim_{x\to 0} \sqrt{-x^2}$ ne postoji.

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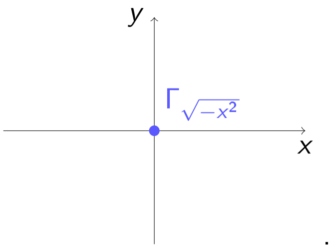
Naime, $\mathcal{D}_{\sqrt{-x^2}} = \{0\}$ pa se $\Gamma_{\sqrt{-x^2}}$ sastoji samo od jedne točke:



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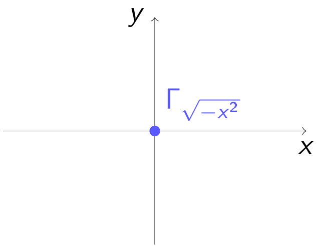

(b) $\lim_{x\to+\infty} \operatorname{arctg}\frac{1}{x}$

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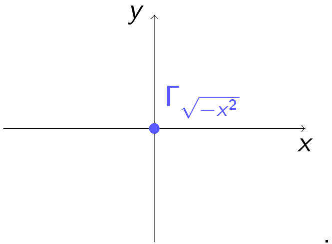

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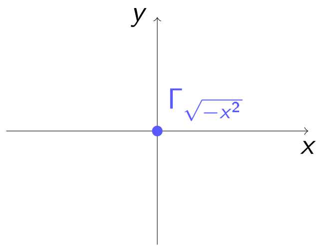

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(b) $\lim_{x\to+\infty} \operatorname{arctg}\dfrac{1}{x} = \operatorname{arctg}\lim_{x\to+\infty}\dfrac{1}{x} = \operatorname{arctg} 0$

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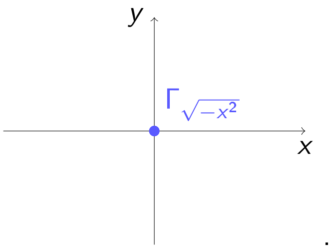

(b) $\lim_{x\to+\infty} \operatorname{arctg}\dfrac{1}{x} = \operatorname{arctg}\lim_{x\to+\infty}\dfrac{1}{x} = \operatorname{arctg} 0 = 0.$

Primjer. $\lim\limits_{x\to 1-} \dfrac{\ln(1-x)}{(x-1)^2}$

Primjer. $\lim\limits_{x \to 1-} \dfrac{\ln \overbrace{(1-x)}^{\to 0+}}{(x-1)^2}$

Primjer. $\lim\limits_{x\to 1-} \dfrac{\ln \overbrace{(1-x)}^{\to 0+}}{(x-1)^2} = \left(\dfrac{-\infty}{0+}\right)$

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$\dfrac{-\infty}{0+}$ je tzv. **određeni oblik** – može se izračunati (i iznosi $-\infty$).

Primjer. $\lim\limits_{x\to 1-} \frac{\ln \overbrace{(1-x)}^{\to 0+}}{(x-1)^2} = \left(\frac{-\infty}{0+}\right) = -\infty.$

$\frac{-\infty}{0+}$ je tzv. **određeni oblik** – može se izračunati (i iznosi $-\infty$). S druge strane,

$$\frac{0}{0}, \qquad \frac{\infty}{\infty}, \qquad (+\infty)+(-\infty), \qquad \infty\cdot 0$$

su tzv. **neodređeni oblici** – ne mogu se jednoznačno odrediti.

Primjer.

- $\lim\limits_{x\to 0} \frac{x}{2x} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \frac{1}{2} = \frac{1}{2}$
- $\lim\limits_{x\to 0} \frac{x^2}{x^4} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \frac{1}{x^2} = +\infty$
- $\lim\limits_{x\to 0} \frac{x^2}{x} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} x = 0$
- $\lim\limits_{x\to 0} \frac{x}{x^2} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \frac{1}{x}$ ne postoji.

Neki određeni oblici

Neka je $a \in \mathbb{R}$. Vrijedi:

- $\dfrac{a}{\infty} = 0$
- $\dfrac{a}{0+} = \begin{cases} +\infty, & \text{ako je } a > 0 \\ -\infty, & \text{ako je } a < 0, \end{cases}$ $\quad \dfrac{a}{0-} = \begin{cases} -\infty, & \text{ako je } a > 0 \\ +\infty, & \text{ako je } a < 0. \end{cases}$
- $\dfrac{+\infty}{0+} = +\infty,$ $\quad \dfrac{+\infty}{0-} = -\infty,$ $\quad \dfrac{-\infty}{0+} = -\infty,$ $\quad \dfrac{-\infty}{0-} = +\infty,$
- $(+\infty) + (+\infty) = +\infty,$ $\quad (-\infty) + (-\infty) = -\infty$
- $(+\infty) \cdot (+\infty) = +\infty,$ $\quad (-\infty) \cdot (-\infty) = +\infty,$ $\quad (+\infty) \cdot (-\infty) = -\infty.$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{(x+1)^2}{x^2+1}$.

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\dfrac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\frac{(x+1)^2}{x^2+1}=\left(\frac{+\infty}{+\infty}\right)$$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} = \left(\frac{+\infty}{+\infty}\right)$$

$$= \lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} &= \left(\frac{+\infty}{+\infty}\right)\\ &= \lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\\ &= \lim_{x\to+\infty} \frac{\left(\frac{x+1}{x}\right)^2}{1+\frac{1}{x^2}}\end{aligned}$$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\left(\frac{x+1}{x}\right)^2}{1+\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\left(1+\frac{1}{x}\right)^2}{1+\frac{1}{x^2}}
\end{aligned}$$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\dfrac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\frac{(x+1)^2}{x^2+1} &= \left(\frac{+\infty}{+\infty}\right)\\
&= \lim_{x\to+\infty}\frac{(x+1)^2}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\\
&= \lim_{x\to+\infty}\frac{\left(\frac{x+1}{x}\right)^2}{1+\frac{1}{x^2}}\\
&= \lim_{x\to+\infty}\frac{\left(1+\frac{1}{x}\right)^2}{1+\frac{1}{x^2}}\\
&= \frac{(1+0)^2}{1+0}
\end{aligned}$$

Zadatak 36(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{(x+1)^2}{x^2+1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{(x+1)^2}{x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\left(\frac{x+1}{x}\right)^2}{1+\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\left(1+\frac{1}{x}\right)^2}{1+\frac{1}{x^2}} \\
&= \frac{(1+0)^2}{1+0} \\
&= 1.
\end{aligned}$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x \to +\infty} \frac{1000x}{x^2 - 1}.$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}$$

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1} = \left(\frac{+\infty}{+\infty}\right)$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1} = \left(\frac{+\infty}{+\infty}\right)$$

$$= \lim_{x\to+\infty} \frac{1000x}{x^2-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x \to +\infty} \frac{1000x}{x^2 - 1}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x \to +\infty} \frac{1000x}{x^2 - 1} &= \left(\frac{+\infty}{+\infty} \right) \\
&= \lim_{x \to +\infty} \frac{1000x}{x^2 - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x \to +\infty} \frac{\frac{1000}{x}}{1 - \frac{1}{x^2}}
\end{aligned}$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{1000x}{x^2-1} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{1000x}{x^2-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\frac{1000}{x}}{1-\frac{1}{x^2}} \\
&= \frac{0}{1-0}
\end{aligned}$$

Zadatak 36(b)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{1000x}{x^2-1}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{1000x}{x^2-1} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{1000x}{x^2-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x\to+\infty} \frac{\frac{1000}{x}}{1-\frac{1}{x^2}} \\
&= \frac{0}{1-0} \\
&= 0.
\end{aligned}$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x \to +\infty} \frac{x^2 - 5x + 1}{3x + 7}.$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2 - 5x + 1}{3x + 7}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{x^2 - 5x + 1}{3x + 7}$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} = \left(\frac{+\infty}{+\infty}\right)$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}.$$

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} = \left(\frac{+\infty}{+\infty}\right)$$

$$= \lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to+\infty} \frac{x-5+\frac{1}{x}}{3+\frac{7}{x}}
\end{aligned}$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to+\infty} \frac{x-5+\frac{1}{x}}{3+\frac{7}{x}} \\
&= \left(\frac{+\infty}{3}\right)
\end{aligned}$$

Zadatak 36(c)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}.$$

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7} &= \left(\frac{+\infty}{+\infty}\right) \\
&= \lim_{x\to+\infty} \frac{x^2-5x+1}{3x+7}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to+\infty} \frac{x-5+\frac{1}{x}}{3+\frac{7}{x}} \\
&= \left(\frac{+\infty}{3}\right) \\
&= +\infty.
\end{aligned}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x \to -\infty} \frac{2x^2 - 3x - 4}{\sqrt{x^4 + 1}}.$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2 - 3x - 4}{\sqrt{x^4 + 1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2 - 3x - 4}{\sqrt{x^4 + 1}} = \left(\frac{+\infty}{+\infty}\right)$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2} = \frac{\sqrt{x^4+1}}{\sqrt{x^4}}$$

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2} = \frac{\sqrt{x^4+1}}{\sqrt{x^4}} = \sqrt{\frac{x^4+1}{x^4}}$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2} = \frac{\sqrt{x^4+1}}{\sqrt{x^4}} = \sqrt{\frac{x^4+1}{x^4}} = \sqrt{1+\frac{1}{x^4}}, \qquad x \in \mathbb{R}\setminus\{0\}.$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}$$

$$= \frac{2-0-0}{\sqrt{1+0}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2} = \frac{\sqrt{x^4+1}}{\sqrt{x^4}} = \sqrt{\frac{x^4+1}{x^4}} = \sqrt{1+\frac{1}{x^4}}, \qquad x \in \mathbb{R}\setminus\{0\}.$$

Zadatak 36(d)

Izračunajte sljedeći limes (ako postoji):

$$\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}.$$

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} &= \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to-\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\ &= \lim_{x\to-\infty} \frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}} \\ &= \frac{2-0-0}{\sqrt{1+0}} \\ &= 2,\end{aligned}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x^4+1}}{x^2} = \frac{\sqrt{x^4+1}}{\sqrt{x^4}} = \sqrt{\frac{x^4+1}{x^4}} = \sqrt{1+\frac{1}{x^4}}, \qquad x\in\mathbb{R}\setminus\{0\}.$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$.

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right)$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}} = \sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}} = \sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$= \sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}, \qquad x \in \langle 0, +\infty\rangle\,.$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}} = \frac{1}{\sqrt{1+\sqrt{0+0}}}$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}} = \sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$= \sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}, \qquad x \in \langle 0, +\infty\rangle .$$

Zadatak 36(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Rješenje. Imamo

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}} = \frac{1}{\sqrt{1+\sqrt{0+0}}} = 1,$$

pri čemu smo u trećoj jednakosti iskoristili da vrijedi

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x^2}}} = \sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$= \sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{\frac{3}{2}}}}}, \qquad x \in \langle 0, +\infty\rangle\,.$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$.

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - 3x + 2}$.

Rješenje. Imamo

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 3x + 2}$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$.

Rješenje. Imamo

$$\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} = \left(\frac{0}{0}\right)$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-1)(x-2)}\end{aligned}$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - 3x + 2}$.

Rješenje. Imamo

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 3x + 2} = \left(\frac{0}{0}\right)$$
$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{(x-1)\cancel{(x-2)}}$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{(x-1)\cancel{(x-2)}}\\ &= \lim_{x\to 2} \frac{x+2}{x-1}\end{aligned}$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{(x-1)\cancel{(x-2)}} \\
&= \lim_{x\to 2} \frac{x+2}{x-1} \\
&= \frac{2+2}{2-1}
\end{aligned}$$

Zadatak 37(a)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{(x-1)\cancel{(x-2)}} \\
&= \lim_{x\to 2} \frac{x+2}{x-1} \\
&= \frac{2+2}{2-1} \\
&= 4.
\end{aligned}$$

Zadatak 37(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 1}$.

Zadatak 37(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \frac{x^2 - 4}{x - 1}$.

Rješenje. Imamo

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 1}$$

Zadatak 37(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-1}$.

Rješenje. Imamo

$$\lim_{x\to 2} \frac{x^2-4}{x-1} = \frac{2^2-4}{2-1}$$

Zadatak 37(b)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to 2} \frac{x^2 - 4}{x - 1}$.

Rješenje. Imamo

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 1} = \frac{2^2 - 4}{2 - 1} = 0.$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \dfrac{x+2}{x^2 - 3x + 2}$.

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\lim_{x\to 2} \frac{x+2}{x^2-3x+2}$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 2} \dfrac{x+2}{x^2 - 3x + 2}$.

Rješenje. Imamo

$$\lim_{x \to 2} \frac{x+2}{x^2 - 3x + 2} = \left(\frac{4}{0}\right)$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 2} \frac{x+2}{x^2-3x+2} &= \left(\frac{4}{0}\right) \\ &= \lim_{x\to 2} \frac{x+2}{(x-1)(x-2)}\end{aligned}$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x+2}{x^2-3x+2} &= \left(\frac{4}{0}\right) \\
&= \lim_{x\to 2} \frac{x+2}{(x-1)(x-2)} \\
&= \left(\frac{4}{1\cdot 0}\right)
\end{aligned}$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2\pm} \frac{x+2}{x^2-3x+2} &= \left(\frac{4}{0}\right) \\
&= \lim_{x\to 2\pm} \frac{x+2}{(x-1)(x-2)} \\
&= \left(\frac{4}{1\cdot 0\pm}\right)
\end{aligned}$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2\pm} \frac{x+2}{x^2-3x+2} &= \left(\frac{4}{0}\right) \\
&= \lim_{x\to 2\pm} \frac{x+2}{(x-1)(x-2)} \\
&= \left(\frac{4}{1\cdot 0\pm}\right) \\
&= \pm\infty,
\end{aligned}$$

Zadatak 37(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x+2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 2\pm} \frac{x+2}{x^2-3x+2} &= \left(\frac{4}{0}\right) \\
&= \lim_{x\to 2\pm} \frac{x+2}{(x-1)(x-2)} \\
&= \left(\frac{4}{1\cdot 0\pm}\right) \\
&= \pm\infty,
\end{aligned}$$

dakle traženi limes ne postoji.

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x + 1}$.

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x + 1}$.

Rješenje. Imamo

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1} = \left(\frac{0}{0}\right)$$

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3+1}{x+1}$.

Rješenje. Imamo

$$\lim_{x \to -1} \frac{x^3+1}{x+1} = \left(\frac{0}{0}\right)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right), \qquad a, b \in \mathbb{R}.$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x + 1}$.

Rješenje. Imamo

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1} = \left(\frac{0}{0}\right)$$
$$= \lim_{x \to -1} \frac{(x + 1)\left(x^2 - x + 1\right)}{x + 1}$$

$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right), \qquad a, b \in \mathbb{R}.$$

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to -1} \frac{x^3+1}{x+1}$.

Rješenje. Imamo

$$\lim_{x\to -1} \frac{x^3+1}{x+1} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to -1} \frac{\cancel{(x+1)}\left(x^2-x+1\right)}{\cancel{x+1}}$$

$$a^3+b^3 = (a+b)\left(a^2-ab+b^2\right), \qquad a, b \in \mathbb{R}.$$

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3+1}{x+1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x \to -1} \frac{x^3+1}{x+1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to -1} \frac{\cancel{(x+1)}\left(x^2-x+1\right)}{\cancel{x+1}} \\
&= \lim_{x \to -1} \left(x^2-x+1\right)
\end{aligned}$$

$$a^3+b^3 = (a+b)\left(a^2-ab+b^2\right), \qquad a, b \in \mathbb{R}.$$

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x + 1}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x \to -1} \frac{x^3 + 1}{x + 1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to -1} \frac{\cancel{(x+1)}\left(x^2 - x + 1\right)}{\cancel{x+1}} \\
&= \lim_{x \to -1} \left(x^2 - x + 1\right) \\
&= (-1)^2 - (-1) + 1
\end{aligned}
$$

$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right), \qquad a, b \in \mathbb{R}.$$

Zadatak 37(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to -1} \dfrac{x^3+1}{x+1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to -1} \frac{x^3+1}{x+1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to -1} \frac{\cancel{(x+1)}\left(x^2-x+1\right)}{\cancel{x+1}} \\
&= \lim_{x\to -1} \left(x^2-x+1\right) \\
&= (-1)^2-(-1)+1 \\
&= 3,
\end{aligned}$$

pri čemu smo u drugoj jednakosti primijenili formulu

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right), \qquad a, b \in \mathbb{R}.$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \left(\frac{0}{0}\right)$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}}\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)}{\sqrt{x}-\sqrt{2}}
\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}}\\ &= \lim_{x\to 2} \frac{\cancel{\left(\sqrt{x}-\sqrt{2}\right)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{\sqrt{x}-\sqrt{2}}}\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2 - \left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{\left(\sqrt{x}-\sqrt{2}\right)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{\sqrt{x}-\sqrt{2}}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right)
\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{\left(\sqrt{x}-\sqrt{2}\right)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{\sqrt{x}-\sqrt{2}}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right) \\
&= \sqrt{2}+\sqrt{2}
\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 1. način. Imamo

$$
\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{\sqrt{x}-\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{\left(\sqrt{x}-\sqrt{2}\right)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{\sqrt{x}-\sqrt{2}}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right) \\
&= \sqrt{2}+\sqrt{2} \\
&= 2\sqrt{2}.
\end{aligned}
$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \left(\frac{0}{0}\right)$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \left(\frac{0}{0}\right)$$
$$= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} \\
&= \lim_{x\to 2} \frac{(x-2)\left(\sqrt{x}+\sqrt{2}\right)}{x-2}
\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{x-2}}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{x-2}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right)
\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{x-2}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right) \\
&= \sqrt{2}+\sqrt{2}
\end{aligned}$$

Zadatak 37(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}-\sqrt{2}}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)}\left(\sqrt{x}+\sqrt{2}\right)}{\cancel{x-2}} \\
&= \lim_{x\to 2} \left(\sqrt{x}+\sqrt{2}\right) \\
&= \sqrt{2}+\sqrt{2} \\
&= 2\sqrt{2}.
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} = \left(\frac{0}{0}\right)$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 1. način. Imamo

$$\begin{aligned} \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} &= \left(\frac{0}{0}\right) \\ &= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{\left(\sqrt{x}\right)^2 - \left(\sqrt{a}\right)^2} \end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\left(\sqrt{x}\right)^2-\left(\sqrt{a}\right)^2} \\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} &= \left(\frac{0}{0} \right) \\
&= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{\left(\sqrt{x}\right)^2 - \left(\sqrt{a}\right)^2} \\
&= \lim_{x \to a} \frac{\cancel{\sqrt{x} - \sqrt{a}}}{\cancel{\left(\sqrt{x} - \sqrt{a}\right)} \left(\sqrt{x} + \sqrt{a}\right)}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\left(\sqrt{x}\right)^2-\left(\sqrt{a}\right)^2} \\
&= \lim_{x\to a} \frac{\cancel{\sqrt{x}-\sqrt{a}}}{\cancel{\left(\sqrt{x}-\sqrt{a}\right)}\left(\sqrt{x}+\sqrt{a}\right)} \\
&= \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\displaystyle\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{\left(\sqrt{x}\right)^2-\left(\sqrt{a}\right)^2} \\
&= \lim_{x\to a} \frac{\cancel{\sqrt{x}-\sqrt{a}}}{\cancel{\left(\sqrt{x}-\sqrt{a}\right)}\left(\sqrt{x}+\sqrt{a}\right)} \\
&= \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}} \\
&= \frac{1}{\sqrt{a}+\sqrt{a}}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. *1. način.* Imamo

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{\left(\sqrt{x}\right)^2 - \left(\sqrt{a}\right)^2} \\
&= \lim_{x \to a} \frac{\cancel{\sqrt{x} - \sqrt{a}}}{\cancel{\left(\sqrt{x} - \sqrt{a}\right)} \left(\sqrt{x} + \sqrt{a}\right)} \\
&= \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} \\
&= \frac{1}{\sqrt{a} + \sqrt{a}} \\
&= \frac{1}{2\sqrt{a}}.
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \left(\frac{0}{0} \right)$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \left(\frac{0}{0} \right)$$

$$= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \\
&= \lim_{x \to a} \frac{x - a}{(x - a)\left(\sqrt{x} + \sqrt{a}\right)}
\end{aligned}$$

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\\ &= \lim_{x\to a} \frac{\cancel{x-a}}{\cancel{(x-a)}\left(\sqrt{x}+\sqrt{a}\right)}\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \\
&= \lim_{x \to a} \frac{\cancel{x - a}}{\cancel{(x - a)} \left(\sqrt{x} + \sqrt{a}\right)} \\
&= \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\\
&= \lim_{x\to a} \frac{\cancel{x-a}}{\cancel{(x-a)}\left(\sqrt{x}+\sqrt{a}\right)}\\
&= \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}}\\
&= \frac{1}{\sqrt{a}+\sqrt{a}}
\end{aligned}$$

Zadatak 37(f)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$.

Rješenje. 2. način. **Racionalizacija:** imamo

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} \\
&= \lim_{x\to a} \frac{\cancel{x-a}}{\cancel{(x-a)}\left(\sqrt{x}+\sqrt{a}\right)} \\
&= \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}} \\
&= \frac{1}{\sqrt{a}+\sqrt{a}} \\
&= \frac{1}{2\sqrt{a}}.
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right)$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3-1}{\left(\sqrt[6]{1+x}\right)^2-1}\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3 - 1}{\left(\sqrt[6]{1+x}\right)^2 - 1} \\
&= \left[\text{Supstitucija:} \quad \begin{array}{r} t = \sqrt[6]{1+x} \\ x \to 0 \;\Rightarrow\; t \to 1 \end{array}\right]
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 0}\frac{\left(\sqrt[6]{1+x}\right)^3-1}{\left(\sqrt[6]{1+x}\right)^2-1}\\
&= \begin{bmatrix}\text{Supstitucija:} & t=\sqrt[6]{1+x}\\ & x\to 0 \ \Rightarrow \ t\to 1\end{bmatrix} = \lim_{t\to 1}\frac{t^3-1}{t^2-1}
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3 - 1}{\left(\sqrt[6]{1+x}\right)^2 - 1} \\
&= \begin{bmatrix} \text{Supstitucija:} & t = \sqrt[6]{1+x} \\ & x \to 0 \ \Rightarrow \ t \to 1 \end{bmatrix} = \lim_{t\to 1} \frac{t^3-1}{t^2-1} \\
&= \lim_{t\to 1} \frac{(t-1)\left(t^2+t+1\right)}{(t-1)(t+1)}
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3 - 1}{\left(\sqrt[6]{1+x}\right)^2 - 1} \\
&= \begin{bmatrix} \text{Supstitucija:} & t = \sqrt[6]{1+x} \\ & x \to 0 \;\Rightarrow\; t \to 1 \end{bmatrix} = \lim_{t\to 1} \frac{t^3-1}{t^2-1} \\
&= \lim_{t\to 1} \frac{\cancel{(t-1)}\left(t^2+t+1\right)}{\cancel{(t-1)}(t+1)}
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3-1}{\left(\sqrt[6]{1+x}\right)^2-1} \\
&= \left[\begin{array}{ll}\text{Supstitucija:} & t=\sqrt[6]{1+x} \\ & x\to 0 \;\Rightarrow\; t\to 1\end{array}\right] = \lim_{t\to 1} \frac{t^3-1}{t^2-1} \\
&= \lim_{t\to 1} \frac{\cancel{(t-1)}\left(t^2+t+1\right)}{\cancel{(t-1)}(t+1)} = \lim_{t\to 1} \frac{t^2+t+1}{t+1}
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 0}\frac{\left(\sqrt[6]{1+x}\right)^3-1}{\left(\sqrt[6]{1+x}\right)^2-1}\\
&= \begin{bmatrix}\text{Supstitucija:} & t=\sqrt[6]{1+x}\\ & x\to 0 \ \Rightarrow\ t\to 1\end{bmatrix} = \lim_{t\to 1}\frac{t^3-1}{t^2-1}\\
&= \lim_{t\to 1}\frac{\cancel{(t-1)}\left(t^2+t+1\right)}{\cancel{(t-1)}(t+1)} = \lim_{t\to 1}\frac{t^2+t+1}{t+1}\\
&= \frac{1^2+1+1}{1+1}
\end{aligned}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 1. način. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\left(\sqrt[6]{1+x}\right)^3 - 1}{\left(\sqrt[6]{1+x}\right)^2 - 1} \\
&= \left[\begin{array}{ll} \text{Supstitucija:} & t = \sqrt[6]{1+x} \\ & x \to 0 \;\Rightarrow\; t \to 1 \end{array}\right] = \lim_{t\to 1} \frac{t^3-1}{t^2-1} \\
&= \lim_{t\to 1} \frac{\cancel{(t-1)}\left(t^2+t+1\right)}{\cancel{(t-1)}(t+1)} = \lim_{t\to 1} \frac{t^2+t+1}{t+1} \\
&= \frac{1^2+1+1}{1+1} = \frac{3}{2}.
\end{aligned}
$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način.

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right)$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-1}{\left(\sqrt[3]{1+x}\right)^3-1}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}$$

$$= \lim_{x\to 0} \underbrace{\frac{\left(\sqrt{1+x}\right)^2 - 1}{\left(\sqrt[3]{1+x}\right)^3 - 1}}_{=\frac{x}{x}=1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}$$

$$= \lim_{x\to 0} \underbrace{\frac{\left(\sqrt{1+x}\right)^2 - 1}{\left(\sqrt[3]{1+x}\right)^3 - 1}}_{=\frac{x}{x}=1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}$$

$$= \lim_{x\to 0} \underbrace{\frac{\left(\sqrt{1+x}\right)^2 - 1}{\left(\sqrt[3]{1+x}\right)^3 - 1}}_{=\frac{x}{x}=1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt[3]{1+x}\right)^2 + \sqrt[3]{1+x}+1}{\sqrt{1+x}+1} = \frac{\left(\sqrt[3]{1+0}\right)^2 + \sqrt[3]{1+0}+1}{\sqrt{1+0}+1}$$

Zadatak 37(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}$$

$$= \lim_{x\to 0} \underbrace{\frac{\left(\sqrt{1+x}\right)^2-1}{\left(\sqrt[3]{1+x}\right)^3-1}}_{=\frac{x}{x}=1} \cdot \frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}+1}{\sqrt{1+x}+1} = \frac{\left(\sqrt[3]{1+0}\right)^2+\sqrt[3]{1+0}+1}{\sqrt{1+0}+1}$$

$$= \frac{3}{2}.$$

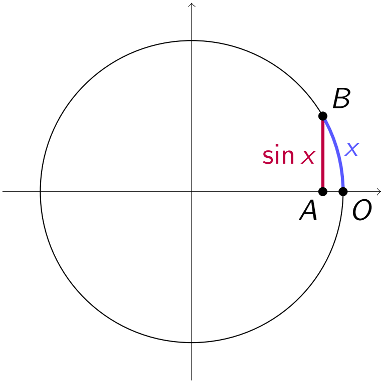
Važan limes 1

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

Važan limes 1

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

Geometrijsko značenje:



$$\lim_{B\to O} \frac{|AB|}{|\overset{\frown}{OB}|} = 1.$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\sin(8x)}{x} = \left(\frac{0}{0}\right)$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\begin{aligned} \lim_{x\to 0} \frac{\sin(8x)}{x} &= \left(\frac{0}{0}\right) \\ &= \lim_{x\to 0} \frac{\sin(8x)}{8x} \cdot 8 \end{aligned}$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(8x)}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0} \frac{\sin(8x)}{8x} \cdot 8\\ &= \begin{bmatrix} t = 8x \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix}\end{aligned}$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin(8x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sin(8x)}{8x} \cdot 8 \\
&= \begin{bmatrix} t = 8x \\ x \to 0 \;\Rightarrow\; t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{\sin t}{t} \cdot 8
\end{aligned}$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin(8x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sin(8x)}{8x} \cdot 8 \\
&= \left[\begin{array}{c} t = 8x \\ x \to 0 \;\Rightarrow\; t \to 0 \end{array}\right] \\
&= \lim_{t\to 0} \frac{\sin t}{t} \cdot 8 \\
&= 1 \cdot 8
\end{aligned}$$

Zadatak 38(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\sin(8x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin(8x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to 0} \frac{\sin(8x)}{8x} \cdot 8 \\
&= \begin{bmatrix} t = 8x \\ x \to 0 \;\Rightarrow\; t \to 0 \end{bmatrix} \\
&= \lim_{t \to 0} \frac{\sin t}{t} \cdot 8 \\
&= 1 \cdot 8 \\
&= 8.
\end{aligned}$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to 0} \frac{\sin(5x)}{\sin(3x)}$.

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{\sin(5x)}{\sin(3x)} = \left(\frac{0}{0}\right)$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(5x)}{\sin(3x)} = \left(\frac{0}{0}\right)$$
$$= \lim_{x\to 0}\frac{\frac{\sin(5x)}{5x}\cdot 5x}{\frac{\sin(3x)}{3x}\cdot 3x}$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{\frac{\sin(5x)}{5x} \cdot 5\cancel{x}}{\frac{\sin(3x)}{3x} \cdot 3\cancel{x}}$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{\sin(5x)}{\sin(3x)} = \left(\frac{0}{0}\right) = \lim_{x \to 0} \frac{\frac{\sin(5x)}{5x} \cdot 5\cancel{x}}{\frac{\sin(3x)}{3x} \cdot 3\cancel{x}} = \lim_{x \to 0} \frac{\frac{\sin(5x)}{5x} \cdot 5}{\frac{\sin 3x}{3x} \cdot 3}$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)} = \left(\frac{0}{0}\right)$$
$$= \lim_{x\to 0} \frac{\frac{\sin(5x)}{5x} \cdot 5\cancel{x}}{\frac{\sin(3x)}{3x} \cdot 3\cancel{x}}$$
$$= \lim_{x\to 0} \frac{\frac{\sin(5x)}{5x} \cdot 5}{\frac{\sin 3x}{3x} \cdot 3}$$
$$= \frac{1 \cdot 5}{1 \cdot 3}$$

Zadatak 38(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sin(5x)}{\sin(3x)}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0}\frac{\sin(5x)}{\sin(3x)} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 0}\frac{\frac{\sin(5x)}{5x}\cdot 5\cancel{x}}{\frac{\sin(3x)}{3x}\cdot 3\cancel{x}}\\
&= \lim_{x\to 0}\frac{\frac{\sin(5x)}{5x}\cdot 5}{\frac{\sin 3x}{3x}\cdot 3}\\
&= \frac{1\cdot 5}{1\cdot 3}\\
&= \frac{5}{3}.
\end{aligned}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} x \cdot \sin \dfrac{\pi}{x}$.

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x\cdot\sin\dfrac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty} x\cdot\sin\frac{\pi}{x} = ((+\infty)\cdot 0)$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x} = ((+\infty)\cdot 0)$$

$$= \lim_{x\to+\infty} x \cdot \frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}} \cdot \frac{\pi}{x}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x\cdot\sin\frac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty} x\cdot\sin\frac{\pi}{x} = ((+\infty)\cdot 0)$$

$$= \lim_{x\to+\infty} \cancel{x}\cdot\frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}}\cdot\frac{\pi}{\cancel{x}}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x\cdot\sin\dfrac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$
\begin{aligned}
\lim_{x\to+\infty} x\cdot\sin\frac{\pi}{x} &= ((+\infty)\cdot 0) \\
&= \lim_{x\to+\infty} \cancel{x}\cdot\frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}}\cdot\frac{\pi}{\cancel{x}} \\
&= \lim_{x\to+\infty} \frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}}\cdot\pi
\end{aligned}
$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} x \cdot \sin \dfrac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$
\begin{aligned}
\lim_{x \to +\infty} x \cdot \sin \frac{\pi}{x} &= ((+\infty) \cdot 0) \\
&= \lim_{x \to +\infty} \cancel{x} \cdot \frac{\sin \frac{\pi}{x}}{\frac{\pi}{x}} \cdot \frac{\pi}{\cancel{x}} \\
&= \lim_{x \to +\infty} \frac{\sin \frac{\pi}{x}}{\frac{\pi}{x}} \cdot \pi \\
&= 1 \cdot \pi
\end{aligned}
$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x \cdot \sin\dfrac{\pi}{x}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x} &= ((+\infty)\cdot 0) \\
&= \lim_{x\to+\infty} \cancel{x} \cdot \frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}} \cdot \frac{\pi}{\cancel{x}} \\
&= \lim_{x\to+\infty} \frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}} \cdot \pi \\
&= 1 \cdot \pi \\
&= \pi.
\end{aligned}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} x \cdot \sin \dfrac{\pi}{x}$.

Rješenje. 2. način.

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x \cdot \sin\dfrac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x} = ((+\infty)\cdot 0)$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x\cdot\sin\dfrac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty} x\cdot\sin\frac{\pi}{x} &= ((+\infty)\cdot 0)\\ &= \begin{bmatrix} t=\frac{\pi}{x} & \leadsto & x=\frac{\pi}{t}\\ x\to+\infty & \Rightarrow & t\to 0\end{bmatrix}\end{aligned}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to +\infty} x \cdot \sin \frac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x \to +\infty} x \cdot \sin \frac{\pi}{x} &= ((+\infty) \cdot 0) \\
&= \left[\begin{matrix} t = \frac{\pi}{x} & \rightsquigarrow & x = \frac{\pi}{t} \\ x \to +\infty & \Rightarrow & t \to 0 \end{matrix} \right] \\
&= \lim_{t \to 0} \frac{\pi}{t} \cdot \sin t
\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x \cdot \sin\frac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x} &= ((+\infty)\cdot 0) \\
&= \begin{bmatrix} t = \frac{\pi}{x} & \rightsquigarrow & x = \frac{\pi}{t} \\ x \to +\infty & \Rightarrow & t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{\pi}{t} \cdot \sin t \\
&= \lim_{t\to 0} \pi \cdot \frac{\sin t}{t}
\end{aligned}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x\cdot\sin\dfrac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty} x\cdot\sin\frac{\pi}{x} &= ((+\infty)\cdot 0)\\ &= \begin{bmatrix} t=\frac{\pi}{x} \quad\rightsquigarrow\quad x=\frac{\pi}{t}\\ x\to+\infty \quad\Rightarrow\quad t\to 0\end{bmatrix}\\ &= \lim_{t\to 0}\frac{\pi}{t}\cdot\sin t\\ &= \lim_{t\to 0}\pi\cdot\frac{\sin t}{t}\\ &= \pi\cdot 1\end{aligned}$$

Zadatak 38(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty} x \cdot \sin\dfrac{\pi}{x}$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty} x \cdot \sin\frac{\pi}{x} &= ((+\infty)\cdot 0) \\
&= \begin{bmatrix} t = \frac{\pi}{x} & \rightsquigarrow & x = \frac{\pi}{t} \\ x \to +\infty & \Rightarrow & t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{\pi}{t} \cdot \sin t \\
&= \lim_{t\to 0} \pi \cdot \frac{\sin t}{t} \\
&= \pi \cdot 1 \\
&= \pi.
\end{aligned}$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\arcsin x}{x}$.

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0} \frac{\arcsin x}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\arcsin x}{x} = \left(\frac{0}{0}\right)$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\arcsin x}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x \to 0} \frac{\arcsin x}{x} &= \left(\frac{0}{0}\right) \\ &= \begin{bmatrix} t = \arcsin x \ \leadsto \ x = \sin t \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix}\end{aligned}$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{\arcsin x}{x} &= \left(\frac{0}{0}\right) \\ &= \begin{bmatrix} t = \arcsin x \ \rightsquigarrow \ x = \sin t \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} \\ &= \lim_{t\to 0} \frac{t}{\sin t}\end{aligned}$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\arcsin x}{x} &= \left(\frac{0}{0}\right) \\
&= \left[\begin{array}{c} t = \arcsin x \;\leadsto\; x = \sin t \\ x \to 0 \;\Rightarrow\; t \to 0 \end{array}\right] \\
&= \lim_{t\to 0} \frac{t}{\sin t} \\
&= \lim_{t\to 0} \frac{1}{\frac{\sin t}{t}}
\end{aligned}$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\arcsin x}{x} &= \left(\frac{0}{0}\right) \\
&= \left[\begin{array}{c} t = \arcsin x \ \rightsquigarrow \ x = \sin t \\ x \to 0 \ \Rightarrow \ t \to 0 \end{array}\right] \\
&= \lim_{t\to 0} \frac{t}{\sin t} \\
&= \lim_{t\to 0} \frac{1}{\frac{\sin t}{t}} \\
&= \frac{1}{1}
\end{aligned}$$

Zadatak 38(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\arcsin x}{x} &= \left(\frac{0}{0}\right) \\
&= \begin{bmatrix} t = \arcsin x \;\; \rightsquigarrow \;\; x = \sin t \\ x \to 0 \;\; \Rightarrow \;\; t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{t}{\sin t} \\
&= \lim_{t\to 0} \frac{1}{\frac{\sin t}{t}} \\
&= \frac{1}{1} \\
&= 1.
\end{aligned}$$

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x}{x}$.

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = \left(\frac{0}{0}\right)$$

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to 0} \frac{\operatorname{tg} x}{x}$.

Rješenje. Imamo

$$\begin{aligned} \lim_{x \to 0} \frac{\operatorname{tg} x}{x} &= \left(\frac{0}{0} \right) \\ &= \lim_{x \to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} \end{aligned}$$

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\operatorname{tg} x}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} \\
&= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}
\end{aligned}$$

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\operatorname{tg} x}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} \\
&= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} \\
&= 1 \cdot \frac{1}{\cos 0}
\end{aligned}$$

Zadatak 38(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\operatorname{tg} x}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} \\
&= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} \\
&= 1 \cdot \frac{1}{\cos 0} \\
&= 1.
\end{aligned}$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \left(\frac{0}{0}\right)$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right)$$

$$\cos(2t) = 1 - 2\sin^2 t, \qquad t \in \mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$\cos(2t) = 1 - 2\sin^2 t, \qquad t \in \mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2}$$

$$\cos(2t) = 1 - 2\sin^2 t, \qquad t \in \mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t = \frac{x}{2} & \rightsquigarrow & x = 2t \\ x \to 0 & \Rightarrow & t \to 0 \end{bmatrix}$$

$$\cos(2t) = 1 - 2\sin^2 t, \qquad t \in \mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t=\frac{x}{2} & \leadsto & x=2t \\ x\to 0 & \Rightarrow & t\to 0 \end{bmatrix} = \lim_{t\to 0} 2\,\frac{\sin^2 t}{(2t)^2}$$

$$\cos(2t) = 1-2\sin^2 t, \qquad t\in\mathbb{R}.$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{1-\cos x}{x^2} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2} \\ &= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t=\frac{x}{2} & \rightsquigarrow & x=2t \\ x\to 0 & \Rightarrow & t\to 0\end{bmatrix} = \lim_{t\to 0} 2\,\frac{\sin^2 t}{(2t)^2} \\ &= \lim_{t\to 0} \frac{1}{2}\left(\frac{\sin t}{t}\right)^2\end{aligned}$$

$$\cos(2t) = 1-2\sin^2 t, \qquad t\in\mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos x}{x^2} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2} \\
&= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t=\frac{x}{2} & \rightsquigarrow & x=2t \\ x\to 0 & \Rightarrow & t\to 0 \end{bmatrix} = \lim_{t\to 0} 2\,\frac{\sin^2 t}{(2t)^2} \\
&= \lim_{t\to 0} \frac{1}{2}\left(\frac{\sin t}{t}\right)^2 = \frac{1}{2}\cdot 1^2
\end{aligned}$$

$$\cos(2t) = 1-2\sin^2 t, \qquad t\in\mathbb{R}.$$

Zadatak 38(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t=\frac{x}{2} & \rightsquigarrow & x=2t \\ x\to 0 & \Rightarrow & t\to 0 \end{bmatrix} = \lim_{t\to 0} 2\,\frac{\sin^2 t}{(2t)^2}$$

$$= \lim_{t\to 0} \frac{1}{2}\left(\frac{\sin t}{t}\right)^2 = \frac{1}{2}\cdot 1^2 = \frac{1}{2},$$

pri čemu smo u drugoj jednakosti iskoristili da vrijedi

$$\cos(2t) = 1 - 2\sin^2 t, \qquad t\in\mathbb{R}.$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos x}{x^2} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2} \\
&= \lim_{x\to 0} 2\,\frac{\sin^2\frac{x}{2}}{x^2} = \begin{bmatrix} t=\frac{x}{2} & \rightsquigarrow & x=2t \\ x\to 0 & \Rightarrow & t\to 0 \end{bmatrix} = \lim_{t\to 0} 2\,\frac{\sin^2 t}{(2t)^2} \\
&= \lim_{t\to 0} \frac{1}{2}\left(\frac{\sin t}{t}\right)^2 = \frac{1}{2}\cdot 1^2 = \frac{1}{2},
\end{aligned}$$

pri čemu smo u drugoj jednakosti iskoristili da vrijedi

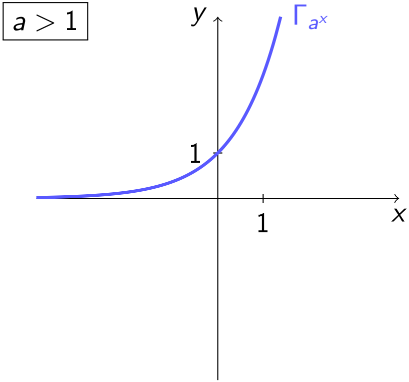
$$\cos(2t) = 1 - 2\sin^2 t, \qquad t\in\mathbb{R}.$$

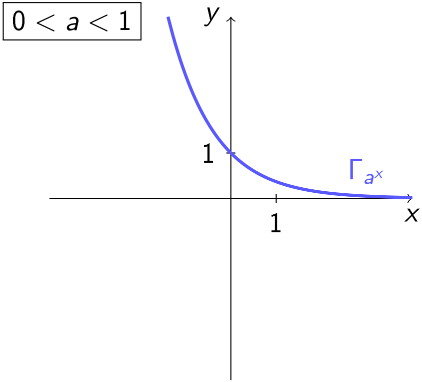
$\rightsquigarrow$ Važan limes:

$$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}.}$$

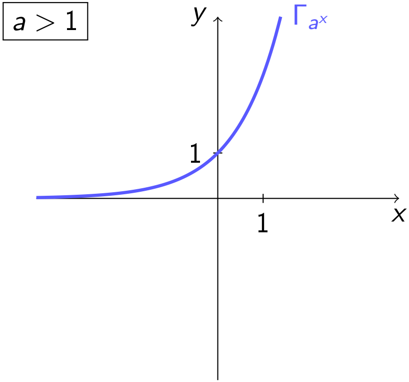
$\lim_{x\to\pm\infty} a^x$

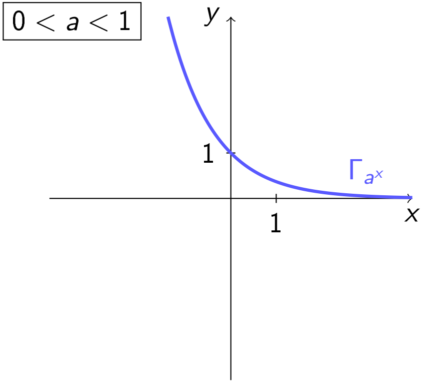
Sjetimo se:





Sjetimo se:





$$\Rightarrow \quad \lim_{x\to-\infty} a^x = \begin{cases} 0, & \text{ako je } a > 1, \\ +\infty, & \text{ako je } 0 < a < 1, \end{cases} \qquad \lim_{x\to+\infty} a^x = \begin{cases} +\infty, & \text{ako je } a > 1, \\ 0, & \text{ako je } 0 < a < 1. \end{cases}$$

$\lim_{x \to c} \varphi(x)^{\psi(x)}$

Kako se, za $c \in \mathbb{R} \cup \{\pm\infty\}$ i $\varphi(x) > 0$, računa limes

$$\lim_{x \to c} \varphi(x)^{\psi(x)}?$$

$\lim_{x\to c} \varphi(x)^{\psi(x)}$

Kako se, za $c \in \mathbb{R} \cup \{\pm\infty\}$ i $\varphi(x) > 0$, računa limes

$$\lim_{x\to c} \varphi(x)^{\psi(x)}?$$

Vrijedi

$$\lim_{x\to c} \varphi(x)^{\psi(x)} = \left(\lim_{x\to c} \varphi(x)\right)^{\lim_{x\to c} \psi(x)}$$

kad god je desna strana definirana, tj. određeni oblik, primjerice:

- A^B sa $A \in \langle 0, +\infty\rangle$ i $B \in \mathbb{R}$
- $A^{+\infty} = \begin{cases} +\infty, & \text{ako je } A > 1, \\ 0, & \text{ako je } 0 < A < 1 \end{cases}$
- $A^{-\infty} = \begin{cases} 0, & \text{ako je } A > 1, \\ +\infty, & \text{ako je } 0 < A < 1. \end{cases}$

Kako se, za $c \in \mathbb{R} \cup \{\pm\infty\}$ i $\varphi(x) > 0$, računa limes

$$\lim_{x\to c} \varphi(x)^{\psi(x)}?$$

Vrijedi

$$\lim_{x\to c} \varphi(x)^{\psi(x)} = \left(\lim_{x\to c} \varphi(x)\right)^{\lim_{x\to c} \psi(x)}$$

kad god je desna strana definirana, tj. određeni oblik, primjerice:

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- $A^{+\infty} = \begin{cases} +\infty, & \text{ako je } A > 1, \\ 0, & \text{ako je } 0 < A < 1 \end{cases}$
- $A^{-\infty} = \begin{cases} 0, & \text{ako je } A > 1, \\ +\infty, & \text{ako je } 0 < A < 1. \end{cases}$

Napomena. 0^0, $(+\infty)^0$ i $1^{\pm\infty}$ su neodređeni oblici.

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \left(\frac{\sin(2x)}{x} \right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{\sin(2x)}{x} = \lim_{x \to 0} \frac{\sin(2x)}{2x} \cdot 2$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\dfrac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2=2$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\sin(2x)}{x} = \lim_{x\to 0} \frac{\sin(2x)}{2x} \cdot 2 = 1 \cdot 2 = 2$$

i

$$\lim_{x\to 0} (1 + x)$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2=2$$

i

$$\lim_{x\to 0}(1+x)=1+0$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\dfrac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2=2$$

i

$$\lim_{x\to 0}(1+x)=1+0=1$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2=2$$

i

$$\lim_{x\to 0}(1+x)=1+0=1$$

pa je

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}=2^1$$

Zadatak 39(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\left(\dfrac{\sin(2x)}{x}\right)^{1+x}$.

Rješenje. Imamo

$$\lim_{x\to 0}\frac{\sin(2x)}{x}=\lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2=1\cdot 2=2$$

i

$$\lim_{x\to 0}(1+x)=1+0=1$$

pa je

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{x}\right)^{1+x}=2^1=2.$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{2x+1}\right)^{x^2}$.

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x+1}{2x+1}\right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\frac{x+1}{2x+1}=\left(\frac{+\infty}{+\infty}\right)$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x+1}{2x+1}\right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\frac{x+1}{2x+1}=\left(\frac{+\infty}{+\infty}\right)=\lim_{x\to+\infty}\frac{x+1}{2x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\dfrac{x+1}{2x+1} \right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{x+1}{2x+1} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \to +\infty} \frac{x+1}{2x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{1 + \frac{1}{x}}{2 + \frac{1}{x}}$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{2x+1}\right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\frac{x+1}{2x+1}=\left(\frac{+\infty}{+\infty}\right)=\lim_{x\to+\infty}\frac{x+1}{2x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to+\infty}\frac{1+\frac{1}{x}}{2+\frac{1}{x}}=\frac{1+0}{2+0}$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\dfrac{x+1}{2x+1} \right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{x+1}{2x+1} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \to +\infty} \frac{x+1}{2x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{1 + \frac{1}{x}}{2 + \frac{1}{x}} = \frac{1+0}{2+0} = \frac{1}{2}$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to +\infty} \left(\frac{x+1}{2x+1} \right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{x+1}{2x+1} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \to +\infty} \frac{x+1}{2x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{1+\frac{1}{x}}{2+\frac{1}{x}} = \frac{1+0}{2+0} = \frac{1}{2}$$

i

$$\lim_{x \to +\infty} x^2$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\dfrac{x+1}{2x+1} \right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{x+1}{2x+1} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \to +\infty} \frac{x+1}{2x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{1+\frac{1}{x}}{2+\frac{1}{x}} = \frac{1+0}{2+0} = \frac{1}{2}$$

i

$$\lim_{x \to +\infty} x^2 = +\infty$$

Zadatak 39(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\frac{x+1}{2x+1} \right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \frac{x+1}{2x+1} = \left(\frac{+\infty}{+\infty} \right) = \lim_{x \to +\infty} \frac{x+1}{2x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{1 + \frac{1}{x}}{2 + \frac{1}{x}} = \frac{1+0}{2+0} = \frac{1}{2}$$

i

$$\lim_{x \to +\infty} x^2 = +\infty$$

pa je

$$\lim_{x \to +\infty} \left(\frac{x+1}{2x+1} \right)^{x^2} = \left(\left(\frac{1}{2} \right)^{+\infty} \right)$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{2x+1}\right)^{x^2}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\frac{x+1}{2x+1}=\left(\frac{+\infty}{+\infty}\right)=\lim_{x\to+\infty}\frac{x+1}{2x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to+\infty}\frac{1+\frac{1}{x}}{2+\frac{1}{x}}=\frac{1+0}{2+0}=\frac{1}{2}$$

i

$$\lim_{x\to+\infty}x^2=+\infty$$

pa je

$$\lim_{x\to+\infty}\left(\frac{x+1}{2x+1}\right)^{x^2}=\left(\left(\frac{1}{2}\right)^{+\infty}\right)=0.$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$

Primjer. Imamo

$$\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}}$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x}=e.}$$

Primjer. Imamo

$$\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}}=\left(1^{\pm\infty}\right)$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$

Primjer. Imamo

$$\begin{aligned}\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\ &= \left[\begin{array}{ccc} t=\frac{1}{x} & \rightsquigarrow & x=\frac{1}{t}\\ x\to 0\pm & \Rightarrow & t\to\pm\infty\end{array}\right]\end{aligned}$$

Jako važni limesi

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$

Primjer. Imamo

$$\begin{aligned}\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}} &= (1^{\pm\infty}) \\ &= \begin{bmatrix} t = \frac{1}{x} & \rightsquigarrow & x = \frac{1}{t} \\ x \to 0\pm & \Rightarrow & t \to \pm\infty \end{bmatrix} \\ &= \lim_{t\to\pm\infty}\left(1+\frac{1}{t}\right)^t\end{aligned}$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$

Primjer. Imamo

$$\begin{aligned}
\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right) \\
&= \begin{bmatrix} t=\frac{1}{x} & \rightsquigarrow & x=\frac{1}{t} \\ x\to 0\pm & \Rightarrow & t\to\pm\infty \end{bmatrix} \\
&= \lim_{t\to\pm\infty}\left(1+\frac{1}{t}\right)^t \\
&= e,
\end{aligned}$$

Vrijedi

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x}=e.}$$

Primjer. Imamo

$$\begin{aligned}
\lim_{x\to 0\pm}(1+x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\
&= \begin{bmatrix} t=\frac{1}{x} & \rightsquigarrow & x=\frac{1}{t}\\ x\to 0\pm & \Rightarrow & t\to\pm\infty\end{bmatrix}\\
&= \lim_{t\to\pm\infty}\left(1+\frac{1}{t}\right)^{t}\\
&= e,
\end{aligned}$$

dakle

$$\boxed{\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e.}$$

Primjer

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\lim_{x \to \pm\infty} \left(1 + \frac{a}{x}\right)^x$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\lim_{x \to \pm\infty} \left(1 + \frac{a}{x}\right)^x = \left(1^{\pm\infty}\right)$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\begin{aligned}
\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^x &= \left(1^{\pm\infty}\right)\\
&= \begin{bmatrix} t = \frac{a}{x} \ \rightsquigarrow \ x = \frac{a}{t} \\ x \to \pm\infty \ \Rightarrow \ t \to 0 \end{bmatrix}
\end{aligned}$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\begin{aligned}
\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^{x} &= \left(1^{\pm\infty}\right) \\
&= \begin{bmatrix} t = \frac{a}{x} \;\rightsquigarrow\; x = \frac{a}{t} \\ x \to \pm\infty \;\Rightarrow\; t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0}\left(1+t\right)^{\frac{a}{t}}
\end{aligned}$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\begin{aligned}
\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^x &= (1^{\pm\infty}) \\
&= \begin{bmatrix} t=\frac{a}{x} \ \rightsquigarrow \ x=\frac{a}{t} \\ x\to\pm\infty \ \Rightarrow \ t\to 0 \end{bmatrix} \\
&= \lim_{t\to 0}(1+t)^{\frac{a}{t}} \\
&= \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^a
\end{aligned}$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$\begin{aligned}
\lim_{x\to\pm\infty} \left(1 + \frac{a}{x}\right)^x &= \left(1^{\pm\infty}\right) \\
&= \begin{bmatrix} t = \frac{a}{x} & \rightsquigarrow & x = \frac{a}{t} \\ x \to \pm\infty & \Rightarrow & t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} (1+t)^{\frac{a}{t}} \\
&= \lim_{t\to 0} \left((1+t)^{\frac{1}{t}}\right)^a \\
&= e^a,
\end{aligned}$$

Primjer

Za svaki $a \in \mathbb{R} \setminus \{0\}$ imamo

$$
\begin{aligned}
\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^x &= \left(1^{\pm\infty}\right) \\
&= \begin{bmatrix} t=\frac{a}{x} & \rightsquigarrow & x=\frac{a}{t} \\ x\to\pm\infty & \Rightarrow & t\to 0 \end{bmatrix} \\
&= \lim_{t\to 0}(1+t)^{\frac{a}{t}} \\
&= \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^a \\
&= e^a,
\end{aligned}
$$

dakle

$$
\boxed{\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^x = e^a.}
$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right)$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right)$$

$$= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right)$$
$$= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= (1^{+\infty})\\ &= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x\\ &= \begin{bmatrix} t=-\frac{2}{x+1} \rightsquigarrow x=-\frac{2}{t}-1\\ x\to+\infty \rightsquigarrow t\to 0\end{bmatrix}\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= \left(1^{+\infty}\right)\\
&= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x\\
&= \begin{bmatrix} t=-\frac{2}{x+1} \ \rightsquigarrow \ x=-\frac{2}{t}-1 \\ x\to+\infty \ \rightsquigarrow \ t\to 0 \end{bmatrix}\\
&= \lim_{t\to 0}(1+t)^{-\frac{2}{t}-1}
\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= \left(1^{+\infty}\right)\\
&= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x\\
&= \begin{bmatrix} t=-\frac{2}{x+1} \rightsquigarrow x=-\frac{2}{t}-1\\ x\to+\infty \rightsquigarrow t\to 0\end{bmatrix}\\
&= \lim_{t\to 0}(1+t)^{-\frac{2}{t}-1} = \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^{-2}\cdot(1+t)^{-1}
\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= \left(1^{+\infty}\right)\\
&= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x\\
&= \begin{bmatrix} t=-\frac{2}{x+1} \;\rightsquigarrow\; x=-\frac{2}{t}-1 \\ x\to+\infty \;\rightsquigarrow\; t\to 0\end{bmatrix}\\
&= \lim_{t\to 0}(1+t)^{-\frac{2}{t}-1} = \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^{-2}\cdot(1+t)^{-1}\\
&= e^{-2}(1+0)^{-1}
\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\displaystyle\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}$.

Rješenje. 1. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x} &= \left(1^{+\infty}\right) \\
&= \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^{x} = \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{x} \\
&= \begin{bmatrix} t=-\frac{2}{x+1} \rightsquigarrow x=-\frac{2}{t}-1 \\ x\to+\infty \rightsquigarrow t\to 0 \end{bmatrix} \\
&= \lim_{t\to 0}(1+t)^{-\frac{2}{t}-1} = \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^{-2}\cdot(1+t)^{-1} \\
&= e^{-2}(1+0)^{-1} \\
&= e^{-2}.
\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. *2. način.*

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\frac{x-1}{x+1} \right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x \to +\infty} \left(\frac{x-1}{x+1} \right)^x = \left(1^{+\infty}\right)$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x-1}{x+1}\right)^x$.

Rješenje. *2. način.* Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^{x}$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{x} = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{array}{c} t=-\frac{2}{x+1} \\ x\to+\infty \;\Rightarrow\; t\to 0 \end{array}\right]$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{array}{c} t=-\frac{2}{x+1} \\ x\to+\infty \;\Rightarrow\; t\to 0 \end{array}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x=\left(1^{+\infty}\right)=\lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$=\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x=\lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}=\left[\begin{array}{c} t=-\frac{2}{x+1} \\ x\to+\infty\ \Rightarrow\ t\to 0\end{array}\right]=\lim_{t\to 0}(1+t)^{\frac{1}{t}}=e$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{array}{c} t=-\frac{2}{x+1} \\ x\to+\infty \ \Rightarrow \ t\to 0 \end{array}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right)$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{matrix} t=-\frac{2}{x+1} \\ x\to+\infty \ \Rightarrow \ t\to 0\end{matrix}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right) = \lim_{x\to+\infty}\frac{-2x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$
$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \begin{bmatrix} t=-\frac{2}{x+1} \\ x\to+\infty \;\Rightarrow\; t\to 0\end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right) = \lim_{x\to+\infty}\frac{-2x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to+\infty}\frac{-2}{1+\frac{1}{x}}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x=(1^{+\infty})=\lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$=\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x=\lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}=\begin{bmatrix} t=-\frac{2}{x+1} \\ x\to+\infty\ \Rightarrow\ t\to 0\end{bmatrix}=\lim_{t\to 0}(1+t)^{\frac{1}{t}}=e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right)=\lim_{x\to+\infty}\frac{-2x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to+\infty}\frac{-2}{1+\frac{1}{x}}=\frac{-2}{1+0}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{matrix} t=-\frac{2}{x+1} \\ x\to+\infty \ \Rightarrow \ t\to 0\end{matrix}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right) = \lim_{x\to+\infty}\frac{-2x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to+\infty}\frac{-2}{1+\frac{1}{x}} = \frac{-2}{1+0} = -2,$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 2. način. Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\left(\frac{x-1}{x+1}-1\right)\right)^x$$

$$= \lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^x = \lim_{x\to+\infty}\left(\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}}\right)^{-\frac{2x}{x+1}}.$$

Kako je

$$\lim_{x\to+\infty}\left(1-\frac{2}{x+1}\right)^{-\frac{x+1}{2}} = \left[\begin{array}{c} t=-\frac{2}{x+1} \\ x\to+\infty \Rightarrow t\to 0\end{array}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to+\infty}\left(-\frac{2x}{x+1}\right) = \lim_{x\to+\infty}\frac{-2x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to+\infty}\frac{-2}{1+\frac{1}{x}} = \frac{-2}{1+0} = -2,$$

slijedi da je

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x = e^{-2}.$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 3. način.

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{x-1}{x+1}\right)^{x}$.

Rješenje. *3. način.* Imamo

$$\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}=\left(1^{+\infty}\right)$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}$.

Rješenje. 3. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x} &= \left(1^{+\infty}\right)\\ &= \lim_{x\to+\infty}\left(\frac{\frac{x-1}{x}}{\frac{x+1}{x}}\right)^{x}\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x}$.

Rješenje. 3. način. Imamo

$$\begin{aligned}
\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^{x} &= \left(1^{+\infty}\right)\\
&= \lim_{x\to+\infty}\left(\frac{\frac{x-1}{x}}{\frac{x+1}{x}}\right)^{x}\\
&= \lim_{x\to+\infty}\frac{\left(1-\frac{1}{x}\right)^{x}}{\left(1+\frac{1}{x}\right)^{x}}
\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 3. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= \left(1^{+\infty}\right)\\ &= \lim_{x\to+\infty}\left(\frac{\frac{x-1}{x}}{\frac{x+1}{x}}\right)^x\\ &= \lim_{x\to+\infty}\frac{\left(1-\frac{1}{x}\right)^x}{\left(1+\frac{1}{x}\right)^x}\\ &= \frac{e^{-1}}{e}\end{aligned}$$

Zadatak 40(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x$.

Rješenje. 3. način. Imamo

$$\begin{aligned}\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right)^x &= (1^{+\infty})\\ &= \lim_{x\to+\infty}\left(\frac{\frac{x-1}{x}}{\frac{x+1}{x}}\right)^x\\ &= \lim_{x\to+\infty}\frac{\left(1-\frac{1}{x}\right)^x}{\left(1+\frac{1}{x}\right)^x}\\ &= \frac{e^{-1}}{e}\\ &= e^{-2}.\end{aligned}$$

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x}$.

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \left(\frac{0}{0}\right)$$

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}\frac{\ln(1+x)}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0}\frac{1}{x}\cdot\ln(1+x)\end{aligned}$$

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{\ln(1+x)}{x} &= \left(\frac{0}{0}\right) \\ &= \lim_{x\to 0} \frac{1}{x} \cdot \ln(1+x) \\ &= \lim_{x\to 0} \ln\left((1+x)^{\frac{1}{x}}\right)\end{aligned}$$

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\ln(1+x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{1}{x} \cdot \ln(1+x) \\
&= \lim_{x\to 0} \ln\left((1+x)^{\frac{1}{x}}\right) \\
&= \ln e
\end{aligned}$$

Zadatak 40(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x \to 0} \frac{\ln(1+x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to 0} \frac{1}{x} \cdot \ln(1+x) \\
&= \lim_{x \to 0} \ln\left((1+x)^{\frac{1}{x}}\right) \\
&= \ln e \\
&= 1,
\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{\ln(1+x)}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0} \frac{1}{x}\cdot \ln(1+x)\\ &= \lim_{x\to 0} \ln\left((1+x)^{\frac{1}{x}}\right)\\ &= \ln e\\ &= 1,\end{aligned}$$

dakle

$$\boxed{\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1.}$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$.

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \left[\begin{array}{c} t = e^x - 1 \;\rightsquigarrow\; e^x = 1 + t \;\rightsquigarrow\; x = \ln(1 + t) \\ x \to 0 \;\Rightarrow\; t \to 0 \end{array} \right]$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \left[\begin{array}{c} t = e^x - 1 \;\leadsto\; e^x = 1 + t \;\leadsto\; x = \ln(1+t) \\ x \to 0 \;\Rightarrow\; t \to 0 \end{array}\right]$$

$$= \lim_{t\to 0} \frac{t}{\ln(1+t)}$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \begin{bmatrix} t = e^x - 1 \ \rightsquigarrow \ e^x = 1 + t \ \rightsquigarrow \ x = \ln(1+t) \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix}$$

$$= \lim_{t\to 0} \frac{t}{\ln(1+t)}$$

$$= \lim_{t\to 0} \frac{1}{\frac{\ln(1+t)}{t}}$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - 1}{x} &= \begin{bmatrix} t = e^x - 1 \ \rightsquigarrow \ e^x = 1 + t \ \rightsquigarrow \ x = \ln(1+t) \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{t}{\ln(1+t)} \\
&= \lim_{t\to 0} \frac{1}{\frac{\ln(1+t)}{t}} \\
&= \frac{1}{1}
\end{aligned}$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - 1}{x} &= \begin{bmatrix} t = e^x - 1 \;\rightsquigarrow\; e^x = 1 + t \;\rightsquigarrow\; x = \ln(1+t) \\ x \to 0 \;\Rightarrow\; t \to 0 \end{bmatrix} \\
&= \lim_{t\to 0} \frac{t}{\ln(1+t)} \\
&= \lim_{t\to 0} \frac{1}{\frac{\ln(1+t)}{t}} \\
&= \frac{1}{1} \\
&= 1,
\end{aligned}$$

Zadatak 40(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{e^x - 1}{x} &= \left[\begin{array}{c} t = e^x - 1 \;\rightsquigarrow\; e^x = 1 + t \;\rightsquigarrow\; x = \ln(1+t) \\ x \to 0 \;\Rightarrow\; t \to 0 \end{array} \right] \\
&= \lim_{t\to 0} \frac{t}{\ln(1+t)} \\
&= \lim_{t\to 0} \frac{1}{\frac{\ln(1+t)}{t}} \\
&= \frac{1}{1} \\
&= 1,
\end{aligned}
$$

dakle

$$
\boxed{\lim_{x\to 0} \frac{e^x - 1}{x} = 1.}
$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sin x}$.

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{e^x - 1}{\sin x} = \left(\frac{0}{0} \right)$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{\frac{e^x-1}{x} \cdot x}{\frac{\sin x}{x} \cdot x}$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \left(\frac{0}{0}\right)$$
$$= \lim_{x\to 0} \frac{\frac{e^x-1}{x} \cdot \cancel{x}}{\frac{\sin x}{x} \cdot \cancel{x}}$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\frac{e^x-1}{x} \cdot \cancel{x}}{\frac{\sin x}{x} \cdot \cancel{x}} = \lim_{x\to 0} \frac{\frac{e^x-1}{x}}{\frac{\sin x}{x}}$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - 1}{\sin x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{e^x-1}{x} \cdot \cancel{x}}{\frac{\sin x}{x} \cdot \cancel{x}} \\
&= \lim_{x\to 0} \frac{\frac{e^x-1}{x}}{\frac{\sin x}{x}} \\
&= \frac{1}{1}
\end{aligned}$$

Zadatak 41(a)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sin x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x \to 0} \frac{e^x - 1}{\sin x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x \to 0} \frac{\frac{e^x - 1}{x} \cdot \cancel{x}}{\frac{\sin x}{x} \cdot \cancel{x}} \\
&= \lim_{x \to 0} \frac{\frac{e^x - 1}{x}}{\frac{\sin x}{x}} \\
&= \frac{1}{1} \\
&= 1.
\end{aligned}
$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}$.

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \left(\frac{0}{0} \right)$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \left(\frac{0}{0} \right)$$

$$a^x = \left(e^{\ln a} \right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \left(\frac{0}{0} \right) = \lim_{x \to 0} \frac{e^{x \ln a} - 1}{x}$$

$$a^x = \left(e^{\ln a} \right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x}$$

$$= \begin{bmatrix} t = x \ln a \rightsquigarrow x = \frac{t}{\ln a} \\ x \to 0 \;\Rightarrow\; t \to 0 \end{bmatrix}$$

$$a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \left(\frac{0}{0} \right) = \lim_{x \to 0} \frac{e^{x \ln a} - 1}{x}$$

$$= \begin{bmatrix} t = x \ln a \ \rightsquigarrow x = \frac{t}{\ln a} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} = \lim_{t \to 0} \frac{e^t - 1}{\frac{t}{\ln a}}$$

$$a^x = \left(e^{\ln a} \right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{e^{x \ln a} - 1}{x} \\
&= \begin{bmatrix} t = x \ln a \ \rightsquigarrow x = \frac{t}{\ln a} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{e^t - 1}{\frac{t}{\ln a}} \\
&= \lim_{t\to 0} \ln a \cdot \frac{e^t - 1}{t}
\end{aligned}$$

$$a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty\rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x} \\
&= \left[\begin{array}{c} t = x\ln a \ \rightsquigarrow x = \frac{t}{\ln a} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{array}\right] = \lim_{t\to 0} \frac{e^t - 1}{\frac{t}{\ln a}} \\
&= \lim_{t\to 0} \ln a \cdot \frac{e^t - 1}{t} = \ln a \cdot 1
\end{aligned}$$

$$a^x = \left(e^{\ln a}\right)^x = e^{x\ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x \to 0} \frac{a^x - 1}{x} &= \left(\frac{0}{0} \right) = \lim_{x \to 0} \frac{e^{x \ln a} - 1}{x} \\
&= \begin{bmatrix} t = x \ln a \ \rightsquigarrow x = \frac{t}{\ln a} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} = \lim_{t \to 0} \frac{e^t - 1}{\frac{t}{\ln a}} \\
&= \lim_{t \to 0} \ln a \cdot \frac{e^t - 1}{t} = \ln a \cdot 1 = \ln a,
\end{aligned}$$

pri čemu smo u drugoj jednakosti iskoristili da vrijedi

$$a^x = \left(e^{\ln a} \right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Zadatak 41(b)

Za $a \in \langle 0, +\infty \rangle$, izračunajte sljedeći limes (ako postoji): $\displaystyle\lim_{x\to 0} \frac{a^x - 1}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{e^{x \ln a} - 1}{x} \\
&= \left[\begin{array}{c} t = x \ln a \ \rightsquigarrow\ x = \frac{t}{\ln a} \\ x \to 0 \ \Rightarrow\ t \to 0 \end{array}\right] = \lim_{t\to 0} \frac{e^t - 1}{\frac{t}{\ln a}} \\
&= \lim_{t\to 0} \ln a \cdot \frac{e^t - 1}{t} = \ln a \cdot 1 = \ln a,
\end{aligned}$$

pri čemu smo u drugoj jednakosti iskoristili da vrijedi

$$a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}, \qquad x \in \mathbb{R}.$$

Dakle,

$$\boxed{\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a.}$$

Zadatak 41(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{8^x - 7^x}{6^x - 5^x}$.

Zadatak 41(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{8^x - 7^x}{6^x - 5^x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{8^x - 7^x}{6^x - 5^x} = \left(\frac{0}{0}\right)$$

Zadatak 41(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{8^x - 7^x}{6^x - 5^x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{8^x - 7^x}{6^x - 5^x} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{\frac{8^x-1}{x} - \frac{7^x-1}{x}}{\frac{6^x-1}{x} - \frac{5^x-1}{x}}$$

Zadatak 41(c)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{8^x - 7^x}{6^x - 5^x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{8^x - 7^x}{6^x - 5^x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{8^x-1}{x} - \frac{7^x-1}{x}}{\frac{6^x-1}{x} - \frac{5^x-1}{x}} \\
&= \frac{\ln 8 - \ln 7}{\ln 6 - \ln 5}.
\end{aligned}$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} (1 + \sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}} = \left(1^{\pm\infty}\right)$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.\end{aligned}$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \left(1^{\pm\infty}\right)$$

$$= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}}$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.\end{aligned}$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \left[\begin{array}{c} t=\sin x \\ x\to 0 \;\Rightarrow\; t\to 0\end{array}\right]$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.\end{aligned}$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \begin{bmatrix} t=\sin x \\ x\to 0 \;\Rightarrow\; t\to 0\end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}}$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \left(1^{\pm\infty}\right)$$

$$= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \begin{bmatrix} t = \sin x \\ x\to 0 \ \Rightarrow \ t\to 0 \end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \left(1^{\pm\infty}\right)$$

$$= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \begin{bmatrix} t = \sin x \\ x \to 0 \;\Rightarrow\; t \to 0 \end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to 0}\frac{\sin x}{x} =$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.\end{aligned}$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \left[\begin{array}{c} t=\sin x \\ x\to 0 \;\Rightarrow\; t\to 0 \end{array}\right] = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to 0}\frac{\sin x}{x} = 1,$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = (1^{\pm\infty})$$

$$= \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \begin{bmatrix} t=\sin x \\ x\to 0 \;\Rightarrow\; t\to 0 \end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to 0}\frac{\sin x}{x} = 1,$$

slijedi da je

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = e^1$$

Zadatak 41(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

Rješenje. Imamo

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \left(1^{\pm\infty}\right) = \lim_{x\to 0}\left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\frac{\sin x}{x}}.$$

Kako vrijedi

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}} = \begin{bmatrix} t=\sin x \\ x\to 0 \;\Rightarrow\; t\to 0 \end{bmatrix} = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$$

i

$$\lim_{x\to 0}\frac{\sin x}{x} = 1,$$

slijedi da je

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = e^1 = e.$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\dfrac{2x+3}{2x+2} \right)^{x+1}$.

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right)$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to +\infty} \left(\frac{2x+3}{2x+2} \right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x \to +\infty} \left(\frac{2x+3}{2x+2} \right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x \to +\infty} \left(1 + \frac{1}{2x+2} \right)^{x+1}$$

$$= \lim_{x \to +\infty} \left(\left(1 + \frac{1}{2x+2} \right)^{2x+2} \right)^{\frac{x+1}{2x+2}}$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2(x+1)}}$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{\cancel{x+1}}{2\cancel{(x+1)}}}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1}=\left(1^{+\infty}\right)=\lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$=\lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}}=\lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{\cancel{x+1}}{2\cancel{(x+1)}}}$$

$$=\lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{1}{2}}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{\cancel{x+1}}{2\cancel{(x+1)}}}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{1}{2}} = \left[\begin{array}{c} t=\frac{1}{2x+2} \rightsquigarrow 2x+2=\frac{1}{t} \\ x\to+\infty \Rightarrow t\to 0 \end{array}\right]$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\dfrac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = \left(1^{+\infty}\right) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{\cancel{x+1}}{2\cancel{(x+1)}}}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{1}{2}} = \begin{bmatrix} t=\frac{1}{2x+2} & \rightsquigarrow & 2x+2=\frac{1}{t} \\ x\to+\infty & \Rightarrow & t\to 0 \end{bmatrix}$$

$$= \lim_{t\to 0}\left(\left(1+t\right)^{\frac{1}{t}}\right)^{\frac{1}{2}}$$

Zadatak 41(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1}$.

Rješenje. Imamo

$$\lim_{x\to+\infty}\left(\frac{2x+3}{2x+2}\right)^{x+1} = (1^{+\infty}) = \lim_{x\to+\infty}\left(1+\frac{1}{2x+2}\right)^{x+1}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{x+1}{2x+2}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{\cancel{x+1}}{2\cancel{(x+1)}}}$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{2x+2}\right)^{2x+2}\right)^{\frac{1}{2}} = \left[\begin{array}{c} t=\frac{1}{2x+2} \rightsquigarrow 2x+2=\frac{1}{t} \\ x\to+\infty \Rightarrow t\to 0 \end{array}\right]$$

$$= \lim_{t\to 0}\left((1+t)^{\frac{1}{t}}\right)^{\frac{1}{2}} = e^{\frac{1}{2}}.$$

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} (1+\operatorname{tg} x)^{\operatorname{ctg} x}$.

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to 0} (1+\operatorname{tg} x)^{\operatorname{ctg} x}$.

Rješenje. Imamo

$$\lim_{x\to 0} (1+\operatorname{tg} x)^{\operatorname{ctg} x} = \left(1^{\pm\infty}\right)$$

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\operatorname{tg} x)^{\operatorname{ctg} x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}(1+\operatorname{tg} x)^{\operatorname{ctg} x} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0}(1+\operatorname{tg} x)^{\frac{1}{\operatorname{tg} x}}\end{aligned}$$

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} (1+\operatorname{tg} x)^{\operatorname{ctg} x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} (1+\operatorname{tg} x)^{\operatorname{ctg} x} &= \left(1^{\pm\infty}\right)\\ &= \lim_{x\to 0} (1+\operatorname{tg} x)^{\frac{1}{\operatorname{tg} x}}\\ &= \begin{bmatrix} t = \operatorname{tg} x \\ x\to 0 \ \Rightarrow \ t\to 0 \end{bmatrix}\end{aligned}$$

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\operatorname{tg}x)^{\operatorname{ctg}x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0}(1+\operatorname{tg}x)^{\operatorname{ctg}x} &= \left(1^{\pm\infty}\right)\\
&= \lim_{x\to 0}(1+\operatorname{tg}x)^{\frac{1}{\operatorname{tg}x}}\\
&= \begin{bmatrix} t=\operatorname{tg}x \\ x\to 0 \;\Rightarrow\; t\to 0\end{bmatrix}\\
&= \lim_{t\to 0}(1+t)^{\frac{1}{t}}
\end{aligned}$$

Zadatak 41(f)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}(1+\operatorname{tg}x)^{\operatorname{ctg}x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0}(1+\operatorname{tg}x)^{\operatorname{ctg}x} &= \left(1^{\pm\infty}\right)\\
&= \lim_{x\to 0}(1+\operatorname{tg}x)^{\frac{1}{\operatorname{tg}x}}\\
&= \begin{bmatrix} t=\operatorname{tg}x \\ x\to 0 \ \Rightarrow \ t\to 0\end{bmatrix}\\
&= \lim_{t\to 0}(1+t)^{\frac{1}{t}}\\
&= e.
\end{aligned}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim_{x \to 0} \frac{\log(1 + 10x)}{x}$.

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje.

$$\lim_{x\to 0} \frac{\log(1+10x)}{x} = \left(\frac{0}{0}\right)$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}\,,\; b \in \langle 0, +\infty\rangle\,,$$

računamo

$$\lim_{x\to 0} \frac{\log(1+10x)}{x} = \left(\frac{0}{0}\right)$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}, \; b \in \langle 0, +\infty\rangle,$$

računamo

$$\lim_{x\to 0} \frac{\log(1+10x)}{x} = \left(\frac{0}{0}\right)$$
$$= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}\,, \;\; b \in \langle 0, +\infty\rangle\,,$$

računamo

$$\begin{aligned}\lim_{x\to 0} \frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right) \\ &= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0} \frac{\ln(1+10x)}{x} \cdot \frac{1}{\ln 10}\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}, \;\; b \in \langle 0, +\infty\rangle,$$

računamo

$$\begin{aligned}\lim_{x\to 0}\frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0}\frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0}\frac{\ln(1+10x)}{x}\cdot\frac{1}{\ln 10}\\ &= \begin{bmatrix} t = 10x \;\rightsquigarrow\; x = \frac{t}{10}\\ x\to 0 \;\Rightarrow\; t\to 0\end{bmatrix}\end{aligned}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}\,,\ \ b \in \langle 0, +\infty\rangle\,,$$

računamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0} \frac{\ln(1+10x)}{x} \cdot \frac{1}{\ln 10} \\
&= \begin{bmatrix} t = 10x \ \leadsto \ x = \frac{t}{10} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{\ln(1+t)}{\frac{t}{10}} \cdot \frac{1}{\ln 10}
\end{aligned}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}, \; b \in \langle 0, +\infty\rangle,$$

računamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0} \frac{\ln(1+10x)}{x} \cdot \frac{1}{\ln 10} \\
&= \begin{bmatrix} t = 10x & \rightsquigarrow & x = \frac{t}{10} \\ x \to 0 & \Rightarrow & t \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{\ln(1+t)}{\frac{t}{10}} \cdot \frac{1}{\ln 10} \\
&= \lim_{t\to 0} \frac{\ln(1+t)}{t} \cdot \frac{10}{\ln 10}
\end{aligned}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}\,,\ \ b \in \langle 0, +\infty\rangle\,,$$

računamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0} \frac{\ln(1+10x)}{x} \cdot \frac{1}{\ln 10} \\
&= \begin{bmatrix} t = 10x & \rightsquigarrow & x = \frac{t}{10} \\ x \to 0 & \Rightarrow & t \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{\ln(1+t)}{\frac{t}{10}} \cdot \frac{1}{\ln 10} \\
&= \lim_{t\to 0} \frac{\ln(1+t)}{t} \cdot \frac{10}{\ln 10} = 1 \cdot \frac{10}{\ln 10}
\end{aligned}$$

Zadatak 41(g)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\log(1+10x)}{x}$.

Rješenje. Koristeći da vrijedi

$$\log_a b = \frac{\log_c b}{\log_c a}, \qquad a, c \in \langle 0, +\infty\rangle \setminus \{1\}\,,\ \ b \in \langle 0, +\infty\rangle\,,$$

računamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\log(1+10x)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\frac{\ln(1+10x)}{\ln 10}}{x} = \lim_{x\to 0} \frac{\ln(1+10x)}{x} \cdot \frac{1}{\ln 10} \\
&= \begin{bmatrix} t = 10x \ \leadsto \ x = \frac{t}{10} \\ x \to 0 \ \Rightarrow \ t \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{\ln(1+t)}{\frac{t}{10}} \cdot \frac{1}{\ln 10} \\
&= \lim_{t\to 0} \frac{\ln(1+t)}{t} \cdot \frac{10}{\ln 10} = 1 \cdot \frac{10}{\ln 10} = \frac{10}{\ln 10}.
\end{aligned}$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$.

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} = \left(\frac{0}{0}\right)$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}\cdot\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\end{aligned}$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= \lim_{x\to 0} \frac{(1+\sin x)-(1-\sin x)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}
\end{aligned}
$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= \lim_{x\to 0} \frac{(1+\sin x)-(1-\sin x)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} \\
&= \lim_{x\to 0} 2 \cdot \frac{\sin x}{x} \cdot \frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}
\end{aligned}
$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= \lim_{x\to 0} \frac{(1+\sin x)-(1-\sin x)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} \\
&= \lim_{x\to 0} 2\cdot\frac{\sin x}{x}\cdot\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= 2\cdot 1\cdot\frac{1}{\sqrt{1+\sin 0}+\sqrt{1-\sin 0}}
\end{aligned}$$

Zadatak 42(a) (**racionalizacija**)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= \lim_{x\to 0} \frac{(1+\sin x)-(1-\sin x)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} \\
&= \lim_{x\to 0} 2 \cdot \frac{\sin x}{x} \cdot \frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} \\
&= 2 \cdot 1 \cdot \frac{1}{\sqrt{1+\sin 0}+\sqrt{1-\sin 0}} = 1.
\end{aligned}$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 0} \dfrac{2 \sin\left(\sqrt{x+1} - 1\right)}{x}$.

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} = \left(\frac{0}{0}\right)$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right) \\ &= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}-1}{x}\end{aligned}$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\end{aligned}$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\sqrt{x+1}-1}{x}\cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{x}{x\left(\sqrt{x+1}+1\right)}
\end{aligned}$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\sqrt{x+1}-1}{x}\cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1}+1\right)}
\end{aligned}
$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} 2\cdot\frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \\
&= \lim_{x\to 0} 2\cdot\frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot\frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1}+1\right)} \\
&= \lim_{x\to 0} 2\cdot\frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot\frac{1}{\sqrt{x+1}+1}
\end{aligned}$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\sqrt{x+1}-1}{x}\cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1}+1\right)} \\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{1}{\sqrt{x+1}+1} \\
&= 2\cdot 1\cdot \frac{1}{\sqrt{0+1}+1}
\end{aligned}
$$

Zadatak 42(b)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 0} \dfrac{2\sin\left(\sqrt{x+1}-1\right)}{x}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 0} \frac{2\sin\left(\sqrt{x+1}-1\right)}{x} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\sqrt{x+1}-1}{x}\cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1}+1\right)}\\
&= \lim_{x\to 0} 2\cdot \frac{\sin\left(\sqrt{x+1}-1\right)}{\sqrt{x+1}-1}\cdot \frac{1}{\sqrt{x+1}+1}\\
&= 2\cdot 1\cdot \frac{1}{\sqrt{0+1}+1}\\
&= 1.
\end{aligned}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty\rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Zadatak 42(c)

Za $a \in \langle 0, +\infty \rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Rješenje.

$$\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} = \left(\frac{0}{0}\right)$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty \rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Rješenje.

$$\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} = \left(\frac{0}{0}\right) = \lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty\rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x-a}$.

Rješenje.

$$a^n - b^n = (a-b)\left(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}\right), \qquad a, b \in \mathbb{R},\ n \in \mathbb{N},$$

$$\lim_{x\to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x-a} = \left(\frac{0}{0}\right) = \lim_{x\to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty\rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x-a}$.

Rješenje.

$$a^n - b^n = (a-b)\left(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}\right), \qquad a, b \in \mathbb{R},\ n \in \mathbb{N},$$

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x-a} = \left(\frac{0}{0}\right) &= \lim_{x\to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n} \\
&= \lim_{x\to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x} - \sqrt[n]{a}\right)\left(\left(\sqrt[n]{x}\right)^{n-1} + \left(\sqrt[n]{x}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}\right)}
\end{aligned}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty\rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Rješenje.

$$a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}\right), \qquad a, b \in \mathbb{R},\ n \in \mathbb{N},$$

$$\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} = \left(\frac{0}{0}\right) = \lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n}$$

$$= \lim_{x \to a} \frac{\cancel{\sqrt[n]{x} - \sqrt[n]{a}}}{\cancel{\left(\sqrt[n]{x} - \sqrt[n]{a}\right)}\left(\left(\sqrt[n]{x}\right)^{n-1} + \left(\sqrt[n]{x}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}\right)}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty \rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\displaystyle\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Rješenje.

$$a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}\right), \qquad a, b \in \mathbb{R},\ n \in \mathbb{N},$$

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} &= \left(\frac{0}{0}\right) = \lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n} \\
&= \lim_{x \to a} \frac{\cancel{\sqrt[n]{x} - \sqrt[n]{a}}}{\cancel{\left(\sqrt[n]{x} - \sqrt[n]{a}\right)}\left(\left(\sqrt[n]{x}\right)^{n-1} + \left(\sqrt[n]{x}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}\right)} \\
&= \frac{1}{\left(\sqrt[n]{a}\right)^{n-1} + \left(\sqrt[n]{a}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}}
\end{aligned}$$

Zadatak 42(c)

Za $a \in \langle 0, +\infty\rangle$ i $n \in \mathbb{N}$, izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to a} \dfrac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a}$.

Rješenje. Koristeći da je

$$a^n - b^n = (a - b)\left(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}\right), \qquad a, b \in \mathbb{R},\ n \in \mathbb{N},$$

računamo

$$\begin{aligned}
\lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x - a} &= \left(\frac{0}{0}\right) = \lim_{x \to a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\left(\sqrt[n]{x}\right)^n - \left(\sqrt[n]{a}\right)^n} \\
&= \lim_{x \to a} \frac{\cancel{\sqrt[n]{x} - \sqrt[n]{a}}}{\cancel{\left(\sqrt[n]{x} - \sqrt[n]{a}\right)}\left(\left(\sqrt[n]{x}\right)^{n-1} + \left(\sqrt[n]{x}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}\right)} \\
&= \frac{1}{\left(\sqrt[n]{a}\right)^{n-1} + \left(\sqrt[n]{a}\right)^{n-2} \cdot \sqrt[n]{a} + \ldots + \left(\sqrt[n]{a}\right)^{n-1}} \\
&= \frac{1}{n\left(\sqrt[n]{a}\right)^{n-1}}.
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 1} \dfrac{\sqrt{x^2 + 3x} - 2x}{x - 1}$.

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\lim_{x \to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} = \left(\frac{0}{0}\right)$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right)\\ &= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x}\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x} \\
&= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x} \\
&= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x} \\
&= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} \\
&= \lim_{x\to 1} \frac{-3x(x-1)}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right) \\
&= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x} \\
&= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} \\
&= \lim_{x\to 1} \frac{-3x\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x^2+3x}+2x\right)}
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x}\\
&= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}\\
&= \lim_{x\to 1} \frac{-3x\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x}{\sqrt{x^2+3x}+2x}
\end{aligned}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2x}{x-1} \cdot \frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x}$$

$$= \lim_{x\to 1} \frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}$$

$$= \lim_{x\to 1} \frac{-3x\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to 1} \frac{-3x}{\sqrt{x^2+3x}+2x}$$

$$= \frac{-3\cdot 1}{\sqrt{1^2+3\cdot 1}+2\cdot 1}$$

Zadatak 42(d)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to1}\dfrac{\sqrt{x^2+3x}-2x}{x-1}$.

Rješenje. Imamo

$$
\begin{aligned}
\lim_{x\to1}\frac{\sqrt{x^2+3x}-2x}{x-1} &= \left(\frac{0}{0}\right)\\
&= \lim_{x\to1}\frac{\sqrt{x^2+3x}-2x}{x-1}\cdot\frac{\sqrt{x^2+3x}+2x}{\sqrt{x^2+3x}+2x}\\
&= \lim_{x\to1}\frac{x^2+3x-4x^2}{(x-1)\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to1}\frac{-3x^2+3x}{(x-1)\left(\sqrt{x^2+3x}+2x\right)}\\
&= \lim_{x\to1}\frac{-3x\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x^2+3x}+2x\right)} = \lim_{x\to1}\frac{-3x}{\sqrt{x^2+3x}+2x}\\
&= \frac{-3\cdot1}{\sqrt{1^2+3\cdot1}+2\cdot1} = -\frac{3}{4}.
\end{aligned}
$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 - 5x + 4} + x \right)$.

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

$$=\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

$$=\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}$$

$$=\lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\ &= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}\\ &= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\end{aligned}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x} \\
&= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.
\end{aligned}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x} \\
&= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.
\end{aligned}$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\ &= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}\\ &= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.\end{aligned}$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}=-\sqrt{\frac{x^2-5x+4}{x^2}}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x} \\
&= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.
\end{aligned}$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}=-\sqrt{\frac{x^2-5x+4}{x^2}}=-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}},\qquad x\in\langle-\infty,0\rangle,$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

$$=\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}$$

$$=\lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}=-\sqrt{\frac{x^2-5x+4}{x^2}}=-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}},\qquad x\in\langle-\infty,0\rangle,$$

pa je

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=\lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}}-1}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\ &= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}\\ &= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.\end{aligned}$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}=-\sqrt{\frac{x^2-5x+4}{x^2}}=-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}},\qquad x\in\langle-\infty,0\rangle,$$

pa je

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=\lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}}-1}=\frac{-5+0}{-\sqrt{1-0+0}-1}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right).$

Rješenje. Imamo

$$\begin{aligned}\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\ &= \lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)\cdot\frac{\sqrt{x^2-5x+4}-x}{\sqrt{x^2-5x+4}-x}\\ &= \lim_{x\to-\infty}\frac{-5x+4}{\sqrt{x^2-5x+4}-x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{\frac{\sqrt{x^2-5x+4}}{x}-1}.\end{aligned}$$

Kako za sve $x\in\langle-\infty,0\rangle$ vrijedi $x=-\sqrt{x^2}$, imamo

$$\frac{\sqrt{x^2-5x+4}}{x}=\frac{\sqrt{x^2-5x+4}}{-\sqrt{x^2}}=-\sqrt{\frac{x^2-5x+4}{x^2}}=-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}},\qquad x\in\langle-\infty,0\rangle,$$

pa je

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=\lim_{x\to-\infty}\frac{-5+\frac{4}{x}}{-\sqrt{1-\frac{5}{x}+\frac{4}{x^2}}-1}=\frac{-5+0}{-\sqrt{1-0+0}-1}=\frac{5}{2}.$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 - 5x + 4} + x \right)$.

Rješenje. 2. način.

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) = ((+\infty)+(-\infty))$$

$$= \left[\begin{array}{c} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{array}\right]$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

$$=\begin{bmatrix} t=-x & \rightsquigarrow x=-t \\ x\to-\infty & \Rightarrow \; t\to+\infty \end{bmatrix}=\lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \left[\begin{array}{c} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{array}\right] = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right) \\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}
\end{aligned}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \left[\begin{array}{c} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{array}\right] = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right) \\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t} \\
&= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}
\end{aligned}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$
\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \begin{bmatrix} t=-x \rightsquigarrow x=-t \\ x\to-\infty \;\Rightarrow\; t\to+\infty \end{bmatrix} = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right) \\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t} \\
&= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}}
\end{aligned}
$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) = ((+\infty)+(-\infty))$$

$$= \begin{bmatrix} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{bmatrix} = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)$$

$$= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}$$

$$= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)=((+\infty)+(-\infty))$$

$$=\begin{bmatrix} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{bmatrix}=\lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)$$

$$=\lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}$$

$$=\lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}}=\lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1}=\lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{\sqrt{t^2}}+1}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\
&= \begin{bmatrix} t=-x \ \rightsquigarrow\ x=-t \\ x\to-\infty \ \Rightarrow\ t\to+\infty \end{bmatrix} = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}\\
&= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{\sqrt{t^2}}+1}\\
&= \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{\frac{t^2+5t+4}{t^2}}+1}
\end{aligned}$$

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty))\\
&= \left[\begin{matrix} t=-x \ \rightsquigarrow\ x=-t \\ x\to-\infty \ \Rightarrow\ t\to+\infty\end{matrix}\right] = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}\\
&= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{\sqrt{t^2}}+1}\\
&= \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{\frac{t^2+5t+4}{t^2}}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{1+\frac{5}{t}+\frac{4}{t^2}}+1}
\end{aligned}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) = ((+\infty)+(-\infty))$$

$$= \left[\begin{array}{c} t=-x \rightsquigarrow x=-t \\ x\to-\infty \;\Rightarrow\; t\to+\infty \end{array}\right] = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)$$

$$= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t}$$

$$= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{\sqrt{t^2}}+1}$$

$$= \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{\frac{t^2+5t+4}{t^2}}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{1+\frac{5}{t}+\frac{4}{t^2}}+1} = \frac{5+0}{\sqrt{1+0+0}+1}$$

Zadatak 42(e)

Izračunajte sljedeći limes (ako postoji): $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right)$.

Rješenje. 2. način. Imamo

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2-5x+4}+x\right) &= ((+\infty)+(-\infty)) \\
&= \begin{bmatrix} t=-x \ \rightsquigarrow x=-t \\ x\to-\infty \ \Rightarrow \ t\to+\infty \end{bmatrix} = \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right) \\
&= \lim_{t\to+\infty}\left(\sqrt{t^2+5t+4}-t\right)\cdot\frac{\sqrt{t^2+5t+4}+t}{\sqrt{t^2+5t+4}+t} \\
&= \lim_{t\to+\infty}\frac{5t+4}{\sqrt{t^2+5t+4}+t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{t}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\frac{\sqrt{t^2+5t+4}}{\sqrt{t^2}}+1} \\
&= \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{\frac{t^2+5t+4}{t^2}}+1} = \lim_{t\to+\infty}\frac{5+\frac{4}{t}}{\sqrt{1+\frac{5}{t}+\frac{4}{t^2}}+1} = \frac{5+0}{\sqrt{1+0+0}+1} = \frac{5}{2},
\end{aligned}$$

pri čemu sedma jednakost vrijedi jer je $t=\sqrt{t^2}$ za sve $t\in[0,+\infty\rangle$.

Oprez!

$$\lim_{x\to 0} \frac{\sin x}{x} \neq \lim_{x\to 0} \frac{\sin 0}{x} = \lim_{x\to 0} 0 = 0.$$